

UDC 624.012.45:620.17

Viktoriiia Kovalchuk^{1*}

¹ Assistant professor, Theoretical and Applied Mechanics Department, State University of Infrastructure and Technologies, 9, Kyrylivska str., Kyiv, 04071, Ukraine. ORCID: <https://orcid.org/0000-0001-7764-8057>

* **Corresponding author:** kovaltchukvv@ukr.net

ANALYTICAL AND NUMERICAL SIMULATION FOR THE FORCE OF ADHESION OF REINFORCING ROD WITH CONCRETE

The purpose of this article is to build a mathematical model for calculating the stress-strain state of structural elements of the road pavement. The model is based on the universal function parameter - the force of adhesion of the reinforcing rod to the concrete along their surface contact. It is proposed to define the specified adhesion force as the solution to the simulated boundary value problem. It is shown that it is expedient to use hyperbolic functions to solve such a problem. For the adhesion force, an analytical dependence is obtained, which is a nonlinear function.

The proposed model of analysis of the stress-strain state of the reinforcement in comparison with other models is relatively simple. The results of modeling using Maple for the analytical dependence of the adhesion force on the length of the reinforcing rod are shown. It is established that the empirical parameter of the system significantly affects the distribution function of normal stresses. The developed theory will allow design engineers to solve the problem of optimizing pavement structures.

Keywords: *differential equations, boundary-value problem, reinforcement bond to concrete, road pavement, stress-strain state.*

Introduction

At the present stage, an important area of research in the transport industry and the road sector is to find ways to extend the life of roads. This requires a corresponding improvement in the method of calculating the strength of the road surface.

Analysis of the results of research and monitoring of road quality shows that the durability of the road significantly depends on the upper layer of the road surface. One of the ways to increase the strength of pavement structures is to use different types of reinforcement for cement concrete pavement. Reinforcement of the upper pavement, and sometimes the lower layers, is used to increase the service life of the road structure.

Bond performance of concrete and reinforcement is of paramount importance in the study of the mechanism of load transfer from concrete to the inner reinforcing bar and vice versa [1]. Therefore, the development of a rational model of the interaction of reinforcement with concrete is now an urgent task for scientists and design engineers.

Analysis of recent research and problem statement

The spread of non-metallic composite reinforcement in the construction of roads in Ukraine determines the relevance of the purposeful experimental study of the adhesion of reinforcement to concrete to ensure the strength, rigidity, and crack resistance of pavement structures. Multi-layered road pavement absorbs loads from different types of vehicles. Similarly, the design of the hard surface of the airfield directly perceives the load from the influence of aircraft, and the influence of

operational equipment. In both cases, the condition of the road surface also largely depends on additional related mechanical factors and climatic conditions [2]. At the same time, full consideration of all these factors in the design of rigid pavement of airfields and roads does not allow to predict their durability at the stage of operation.

The gradual destruction of the multilayer road pavement is due to mechanical loads, temperature changes, changes in humidity etc. To prevent the consequences of such exposure to aggressive environments and increase the service life of the road structure using reinforcement of the upper road pavement, and sometimes the lower layers of the road pavement. In appropriate concrete structures, composite reinforcement is considered a promising alternative to steel reinforcement.

Wide use of composite reinforcement related to a unique combination of high strength, corrosion resistance, and low cost. The glass fiber reinforcement bars are widely used in structural engineering as reinforcement of concrete structures [4]. Compared to steel reinforcement, fiberglass reinforcement bars have such advantages as low weight, high strength, electromagnetic neutrality, corrosion resistance and low cost. All these benefits are important factors to increase durability and to reduce of the cost of technical maintenance concrete structures of the pavement, reinforced with this type of rods [5].

The decisive factor to ensure the strength of the pavement structure is the mechanical interaction of reinforcement with concrete, which depends on the size and shape of the cross-section, the profile of the outer surface of the reinforcement, and as well as the mechanical properties of the materials. Quite a lot of experimental and theoretical work is devoted to the interaction of reinforcement with concrete [1 – 16]. Testing of composite elements under direct tension is of fundamental importance to reveal the tension load response and cracking behavior of reinforced concrete [3, 8].

An experimental comprehensive study of the impact of various indicators is carried out by studying the problem of extracting the reinforcing bar from concrete [2]. Recently, not only experimental and numerical studies have become important for this task. Analytical models are also becoming an important component of the general theory of adhesion reinforcement with concrete [3].

The purpose and tasks of the study

The aim of the work is to build a mathematical model of the mechanism of load transfer from concrete to the inner reinforcing bar. The study is aimed at obtaining an analytical dependence for the force of adhesion of the reinforcing bar with concrete along their surface contact.

The second stage of this study is to conduct numerical experiments to analyze the effect of model parameters on the distribution of normal forces in the reinforcing bar and concrete.

Mathematical model of the problem of adhesion of concrete and reinforcement (analytical approach)

Forces of adhesion create a complex stress-strain state in armature and concrete. Accuracy determination of the main parameters of adhesion in reinforcement bond to concrete element depends to a large extent on taking into account real modes of loads and conditions of operation of structures, as well as the correct choice of calculation scheme [16].

Consider the problem of the stress-strain state of a reinforced concrete rod under the conditions of pulling out the reinforcement. Under certain assumptions, the process of interaction of reinforcement with concrete can be analyzed by the behavior of the threaded connection of two parts: a concrete sleeve with an internal thread and a reinforcement rod with a corresponding external thread.

Whenever an external load is applied to concrete, the reinforcing bar receives part of the load through a load transfer mechanism from concrete to steel or composite armature [1]. When a tensile force is applied to the reinforcing bar, it develops stress components parallel and perpendicular to the contact surface. Consider the distribution of normal force directed parallel to the rod.

To analyze the stress-strain state of the rod under certain assumptions, consider a simplified calculation scheme: a reinforcing rod that interacts with a concrete block on a section of length l . To

build a mathematical model, the calculation scheme of the reinforcement element of such a short length dx is accepted that the force per unit length of the reinforcing rod in contact with concrete can be considered proportional to the mutual movement of the rod and sleeve.

Two identical oppositely directed tensile forces \vec{F} act on the rod and on the block. To determine the analytical dependence of the distribution of normal forces \vec{N}_a acting in the reinforcing rod in the specified anchoring area, consider the design element of the rod, the length dx of which is assumed sufficiently small. An analytical model of the bond reinforcement with concrete consists of a second-order linear differential equation [6]:

$$\frac{d^2 N_a}{dx^2} - k^2 N_a = -c \cdot \frac{F}{E_b A_b}, \quad (1)$$

where c – the empirical proportionality factor,

A_b – the cross-sectional area of the concrete block,

E_b – elastic modulus of concrete.

When solution the equation (1), we take into account the boundary conditions:

$$N_a(0) = 0, \quad N_a(l) = F. \quad (2)$$

The general solution of the linear inhomogeneous differential equation (1) we can search in the standard form:

$$N_a(x) = B_1 \exp(kx) + B_2 \exp(-kx) + f_1(x). \quad (3)$$

Then the solution of the boundary-value problem (1)–(2) has the form:

$$N_a(x) = B_1 \exp(kx) + B_2 \exp(-kx) + \frac{c}{k^2} \cdot \frac{F}{E_b A_b}. \quad (4)$$

Here we have the following expressions for constants B_1 and B_2 :

$$B_1 = \frac{F}{2 \operatorname{sh}(kl)} \left(1 + \frac{c}{k^2} \cdot \frac{\exp(-kl) - 1}{E_b A_b} \right),$$

$$B_2 = -\frac{F}{2 \operatorname{sh}(kl)} \left(1 + \frac{c}{k^2} \cdot \frac{\exp(kl) - 1}{E_b A_b} \right).$$

You can also write the general solution of equation (1) in another form:

$$N_a(x) = C_1 \operatorname{sh}(kx) + C_2 \operatorname{ch}(kx) + \frac{c}{k^2} \cdot \frac{F}{E_b A_b}. \quad (5)$$

The integration constants C_1 and C_2 are determined from the boundary conditions (2) of the boundary value problem:

$$C_1 = \frac{F}{\operatorname{sh}(kl)} \left(1 + \frac{c}{k^2} \cdot \frac{\operatorname{ch}(kl) - 1}{E_b A_b} \right), \quad C_2 = -\frac{c}{k^2} \cdot \frac{F}{E_b A_b}.$$

The given formulae show that expression (5) is a more compact form for the analytical solution of the problem.

The integration constants C_1 and C_2 in expressions (5) or integration constants B_1 and B_2 in expressions (4) are determined from the boundary conditions (2) of the boundary value problem. These constants depend on the geometric and mechanical characteristics of the structural material of the reinforcing rod and the concrete block. Subsequent calculations showed that the empirical coefficient c also significantly affects the solution of the boundary-value problem (1) – (2).

Formula (5) gives the expression for the adhesion force in the General case. Calculations have shown that for the case of fiberglass fittings it is possible to take an approximate dependence in the form:

$$N_a(x) = F \cdot \frac{\operatorname{sh}(kx)}{\operatorname{sh}(kl)}. \tag{6}$$

If the variable x runs the value from the interval $x \in [0, l]$, then in the area of interaction of the reinforcing rod with the concrete block, the normal force N_a in the rod becomes values from 0 to F . Since for the force in the concrete block we have $N_b = F - N_a$, then when x changes from 0 to l , the forces N_b in the block change from F to 0.

Simulation results

According to the constructed mathematical model and analytical dependence for the distribution of normal forces, we analyze the influence of system parameters. In particular, we consider the influence of mechanical and geometric characteristics of the reinforcing bar and cement block on the stress-strain state of the reinforcement.

Here we performed several numerical simulations using the Maple computer mathematics system. Numerical simulations were performed for the experimental sample of the reinforcing rod and concrete block with the parameters listed in Table 1.

Table 1. List of Parameters for numerical simulation

Symbol	Description	Value/Unit
E_a	elastic modulus of reinforcing (fiberglass)	45 000 MPa
E_b	elastic modulus of concrete	30 000 MPa
d	nominal diameter of reinforcing rod	14 mm
l	bonded length ($l = 5d$)	70 mm
b	side of the cross section of the concrete block	150 mm
F	external force	80 kN

The comparative analysis of the obtained experimental data shows that the magnitude of the normal forces in the reinforced rod significantly depends on the choice of the empirical coefficient c . The graph (Fig. 1), constructed for fiberglass reinforcement, shows that for $c = 2 \cdot 10^5$ the dependence $N_a(x)$ is significantly nonlinear.

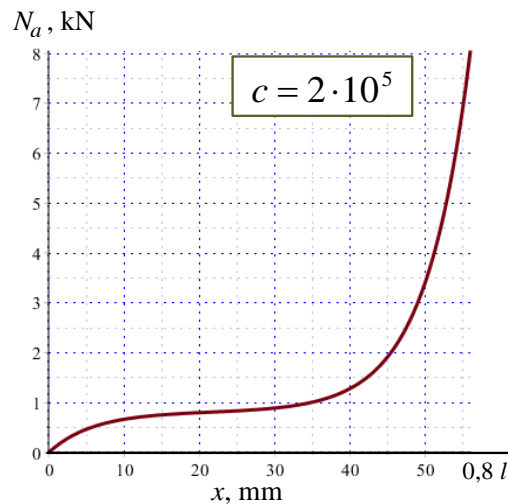


Figure 1. Distribution of normal forces in a reinforced rod

The distribution of normal forces in the reinforced rod at $c = 2 \cdot 10^3$ is almost linear (Fig. 2, a). To show the evolution of the nonlinearity of the normal force function $N_a(x)$ depending on the empirical coefficient, a graph at $c = 2 \cdot 10^4$ is also constructed (Fig. 2, b).

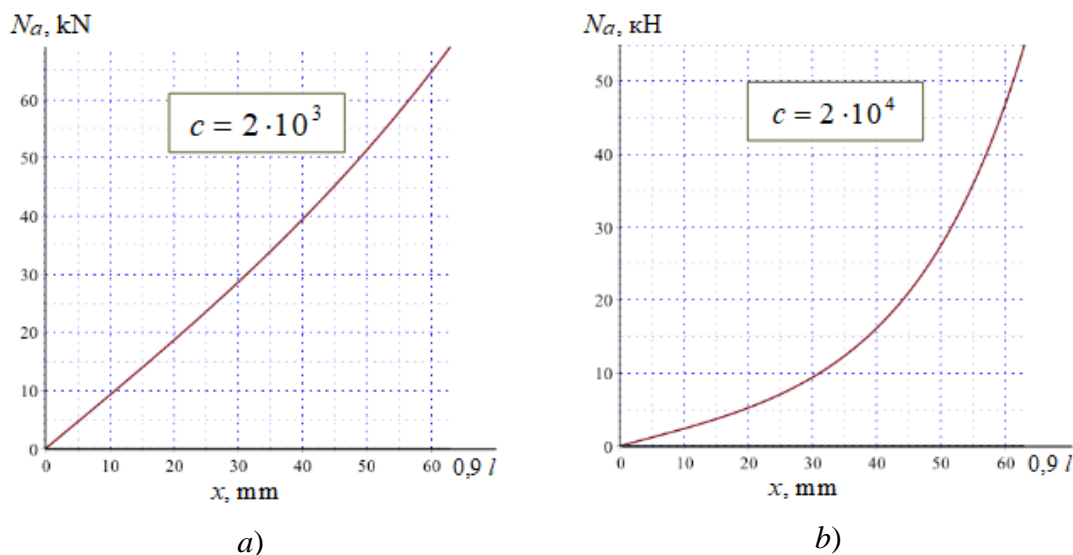


Figure 2. Influence of the empirical coefficient on the distribution of normal forces

The graph in Fig. 1 has an inflection point. This point will correspond to the extreme point on the graph for the intensity $q(x)$ of normal stresses. The intensity of normal stresses can be obtained analytically from expression (5) or (6) by differentiation:

$$q = \frac{dN_a}{dx}$$

If we take into account the exact analytical solution (5) of the boundary value problem, we have the following expression:

$$q(x) = \frac{Fk}{\text{sh}(kl)} \left(\left(1 - \frac{c}{k^2 E_b A_b} \right) \text{ch}(kx) + \frac{c}{k^2 E_b A_b} \text{ch}(k(l-x)) \right)$$

According to the approximate formula (6) we have:

$$q(x) = Fk \frac{\text{ch}(kx)}{\text{sh}(kl)}$$

Therefore, for composite reinforcement (at certain values of geometric and mechanical parameters) the function $q(x)$ has a simpler form.

For the fiberglass reinforcing rod, it is analyzed at what limits of accuracy expression (6) can be an approximation for the function of adhesion force (5). With the numerical parameters shown in Table 1, a graph of the exact analytical solution of the boundary value problem (1) – (2) was constructed. For comparison, a graph of the main part of this solution was also constructed. In Figure 3, curve 1 is constructed for analytical dependence (5), and for curve 2, formula (56) is used.

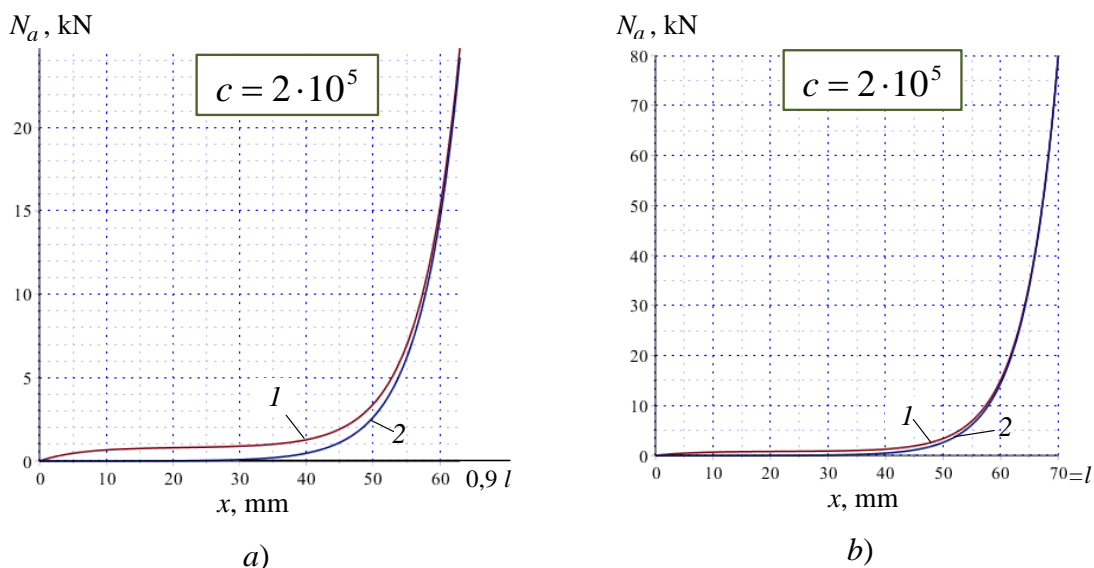


Figure 3. Normal force (curve 1) and thereof main part (curve 2) for a fiberglass reinforcing rod

Comparative analysis of Fig. 3, a) and Fig. 3, b) shows that the differences between (5) and (6) are insignificant and are observed only for small values of x . This means that only the main part (6) of the function $N_a(x)$ can be used for further studies of the stress-strain state of the reinforcement.

Similar studies have also been carried out in the case of steel reinforcement. In the constructed graphs, the difference between curves 1 and 2 is more significant. This means that an accurate analytical solution (5) must be used to analyze the stress-strain state of a steel reinforcing bar.

4. Conclusions

Finding ways to improve the reliability and durability of roads at the present stage is an important area of research in the transport industry and the road sector. At the same time, the task of improving the method of calculations of the arrangement of the upper layer of the pavement with the use of high-strength cement concrete is urgent. In this study, a mathematical model of the interaction of reinforcement with concrete is proposed.

The analytical approach to the analysis of the stress-strain state of the reinforcement in the conditions of interaction with high-strength concrete allows obtaining the dependence of the force of engagement of the reinforcement with concrete based on a simplified model. This functional dependence is nonlinear. The advantage of the analytical approach is the avoidance of numerical errors. The developed theory will allow design engineers to solve the problem of optimizing pavement structures.

REFERENCES

1. Kabir, M. R., & Islam, M. M. (2014). Bond stress behavior between concrete and steel rebar: Critical investigation of pull-out test via Finite Element Modeling. *Int J Civ Struct Eng*, 5(1), 80–90.
2. Mirnenko, V. I., Dulenko, D. I., & Dyptan, V. P. (2017). Modeli ta metodi ocinki napruzhenno-deformovanogo stanu zhorstkich pokryttiv aerodromiv viyskovogo pryznachennja na osnovy kincevo-elementnogo analizu [Mathematical simulation methodology for prediction of the airfield hard surface condition]. *Modern Information Technologies in the Sphere of Security and Defence*, 30 (3), 146–151 [in Ukrainian].
3. Romashko-Maistruk, O. V. (2020). Modelyuvannya zcheplennya armatury z betonom v zalizobetonnyh elementach [Modeling of reinforcement-concrete adhesion in reinforced concrete elements]. *Zbirnyk naukovykh praz Ukrainyiskoho derzhavnoho universytetu zaliznychnoho transportu – Collected scientific works of Ukrainian State University of Railway Transport*, 190, 37-45 [in Ukrainian]. <https://doi.org/10.18664/1994-7852.190.2020.213925> .
4. Benin, A., Bogdanova, G., Semenov, S. (2014). Experimental study and mathematical modeling of bond of different types winding glass-plastic reinforcement with concrete. *Applied Mechanics and Materials*, 617, 215–220. <https://doi.org/10.4028/www.scientific.net/AMM.617.215> .
5. Benin, A., Semenov, S., & Bogdanova, E. (2017). Influence of Long-Term Exposure in the Concrete of FRP Rebars on Bond Characteristics. *Solid State Phenomena*, 263, 3-6. <https://doi.org/10.4028/www.scientific.net/SSP.263.3>.
6. Kiyashko, V., Kosarchuk, V., Chausov, M., Agarcov, O., & Kovalchuk, V. (2020). Napruzhenno-deformovaniy stan kompozitnoyi armatury za umov vzayemodiyi z visokomicnym betonom: analitichni rozrachunki [Stress-strain state of composite reinforcement under the conditions of interaction with high-strength concrete: analytical approach]. *Ecological Sciences*, 2(29), 100–104 [in Ukrainian]. <https://doi.org/10.32846/2306-9716/2020.eco.2-29.2.16> .
7. Kabir, M. R., Islam, M. M. & Chowdhury, M. A. (2015). Bond stress-slip behavior between concrete and steel rebar via pullout test: experimental and finite element analysis. *First International Conference on Advances in Civil Infrastructure and Construction Materials*, Retrieved from <https://cutt.ly/yJa4A5t> .
8. Gribniak, V., Jakubovskis, R., Rimkus, A., Ng, P. L., & Hui, D. (2018). Experimental and numerical analysis of strain gradient in tensile concrete prisms reinforced with multiple bars. *Construction and Building Materials*, 187, 572-583. <https://doi.org/10.48550/arXiv.1906.00234> .
9. Liu Z., Yue Q., Li R., & Chen X. (2020). Experimental study and modeling of bond of carbon-fiber-reinforced polymer grids to polymer mortar at room and elevated temperatures. *Advances in Structural Engineering*, 23(8), 1644-1655. DOI: <https://doi.org/10.1177/1369433219899784>
10. Elgabbas, F., Vincent, P., Ahmed, E.A., & Benmokrane, B. (2016). Experimental testing of basalt-fiber-reinforced polymer bars in concrete beams. *Composites Part B: Engineering*, 91, 205-218. <https://doi.org/10.1016/j.compositesb.2016.01.045>.
11. Ling, J., Wei, F., Zhao, H., Tian, Y., & Han, B. (2019). Analysis of airfield composite pavement responses using full-scale accelerated pavement testing and finite element method. *Construction and Building Materials*, 212, 596-606. <https://doi.org/10.1016/j.conbuildmat.2019.03.336>.
12. Zheng, Y., Zhou, L., Xia, L., Luo, Y., & Taylor, S. E. (2018). Investigation of the behaviour of SCC bridge deck slabs reinforced with BFRP bars under concentrated loads. *Engineering Structures*, 171, 500-515. <https://doi.org/10.1016/j.engstruct.2018.05.105>.

13. Fakoor, M., & Nematzadeh, M. (2021). Evaluation of post-fire pull-out behavior of steel rebars in high-strength concrete containing waste PET and steel fibers: Experimental and theoretical study. *Construction and Building Materials*, 299, 123917. <https://doi.org/10.1016/j.conbuildmat.2021.123917>.

14. Hu, A., Liang, X., & Shi, Q. (2020). Bond characteristics between high-strength bars and ultrahigh-performance concrete. *Journal of Materials in Civil Engineering*, 32(1), 04019323. [http://dx.doi.org/10.1061/\(ASCE\)MT.1943-5533.0002919](http://dx.doi.org/10.1061/(ASCE)MT.1943-5533.0002919).

15. Qasem, A., Sallam, Y. S., Eldien, H. H., & Ahangarn, B. H. (2020). Bond-slip behavior between ultra-high-performance concrete and carbon fiber reinforced polymer bars using a pull-out test and numerical modelling. *Construction and Building Materials*, 260, 119857. <https://doi.org/10.1016/j.conbuildmat.2020.119857>.

16. Kolchunov, V. I., Yakovenko, I. A., & Dmitrenko, E. A. (2016). Konechno-elementnoye modelirovaniye nelineynoy ploskoy zadachi scepheniya betona i armatury v PK Lira-SAPR [Finite-element modeling of the nonlinear plane problem of adhesion of concrete and reinforcement in the Lira-CAD PC]. *Promyslove budivnytstvo ta inzhenerni sporudy – Industrial construction and engineering structures*, 3, 6-15 [in Russian]. Retrieved from http://nbuv.gov.ua/UJRN/Pbis_2016_3_3.

Вікторія Ковальчук¹

¹ Доцент, Кафедра теоретичної і прикладної механіки, Державний університет інфраструктури та технологій, вул. Кирилівська, 9, м. Київ, 04071, Україна

АНАЛІТИЧНЕ ТА ЧИСЕЛЬНЕ МОДЕЛЮВАННЯ СИЛИ ЗЧЕПЛЕННЯ АРМАТУРНОГО СТЕРЖНЯ З БЕТОНОМ

Метою даної статті є побудова математичної моделі для розрахунку напружено-деформованого стану конструктивних елементів дорожнього покриття автомобільних доріг. Модель базується на універсальному функціональному параметрі – силі зчеплення арматурного стержня з бетоном уздовж їх поверхневого контакту. Запропоновано визначати задану силу зчеплення як розв'язок змодельованої крайової задачі. Показано, що для розв'язання такої задачі доцільно використовувати гіперболічні функції. Для сили зчеплення отримано аналітичну залежність, яка є нелінійною функцією.

Запропонована модель аналізу напружено-деформованого стану арматури в порівнянні з іншими моделями є відносно простою. Наведено результати моделювання за допомогою Maple для аналітичної залежності сили зчеплення від довжини арматурного стержня. Встановлено, що емпіричний параметр системи суттєво впливає на функцію розподілу нормальних напружень. Розроблена теорія дозволить інженерам-проектувальникам вирішувати задачу оптимізації конструкцій дорожнього покриття.

Ключові слова: диференціальні рівняння, крайова задача, зчеплення арматури з бетоном, дорожнє покриття, напружено-деформований стан.