Models of criterion evaluation of the image processing systems effectiveness

The criteria used to evaluate the effectiveness of image processing systems are investigated in the article. The requirements for performance criteria are analyzed. Private criteria which are used for image processing systems are selected and justified. Such parameters as performance, system cost, hardware costs characterize the system itself and depend on its specific type. It was shown that the information capacity, the probability of completing the task, and the accuracy of the image processing algorithm are the main criteria that characterize the quality of the processing method. It is shown that such a reliability criterion as normalized root mean square error best meets the requirements of efficiency criteria. Criteria models that are based on the normalized root mean square error in relation to discrete images have been studied. The simulation results and obtained dependences of cost functions on the speed of system information processing are given. The methodology for choosing a generalized criterion, which characterizes not only the information processing system, but also the methods used to implement this system was found. We obtained a generalized cost criterion, which arguments are the accuracy of system operation, speed of operation, and cost advantages.

Keywords: Efficiency criterion, image processing, normalized root mean square error.

Introduction. Synthesis of any system, in particular a image processing system, should begin with choosing and justifying the criteria for evaluation of efficiency, quality and optimization. At the same time, it is necessary to choose a criterion that would allow synthesizing an optimal process and a system regarding the most important indicators of quality efficiency. These indicators should in the first place include the following:
- Problem solving probability or system reliability;
- Information capacity;
- Speed;
- Volume and weight, complexity and cost;
- Accuracy of work and controllability of the system itself;
- Noise immunity.

In addition to these requirements, the criterion should possess certain constructability in order to easily evaluate its numerical value, which would allow calculating the efficiency of not only the process, device and the system itself in its proximity to potential perfection, but also comparatively similar devices, processes and systems in cumulation.

Propositions and consequences formulated in this research allows deriving a generalized functional-statistical efficiency evaluation criterion that satisfies all above-mentioned requirements. Main properties of the generalized criterion and particular criteria derived from the general, based on the maximum total utility at minimum cost are also described.
Analysis of recent research and problem statement. The concept of quality efficiency and optimality occupies an important place in theories about information systems in general and image processing systems, as well as for evaluating of information processing methods used in these systems. The efficiency of the system can be considered as the benefit received as a result of its use, attributed to the costs of its development and operation $E_s$ [1].

Criteria are used to evaluate efficiency. A criterion is an indicator or a measure of the effectiveness of the quality and optimality of the system. Criteria can be private or general.

We have to deal with the justification and selection of performance evaluation and optimization criteria at all stages of system modeling. The difficulty of this task lies in the fact that for complex systems it is not possible to choose a single criterion for evaluating efficiency and optimality [2-4]. Therefore, it is necessary to solve the problem of multi-criteria ranking or optimization, the main thing in this is the choice and justification of the compromise principle, that is, the generalized scalar criterion.

When choosing criteria, it is necessary to take into account a lot of requirements [1, 5, 6]:

- the criterion should directly characterize efficiency, reflect objective reality;
- the criterion must have a physical interpretation;
- the criterion should be simple and easy to calculate;
- the value of the criterion must be within certain limits, for example $0 \leq E_s \leq 1$, that means that the criterion of an inefficient system is equal to zero, and the criterion of an ideal system is equal to one;
- the criterion shouldn’t have dimensionality, that is, it should be normalized;
- the criterion must be theoretical, that is, formulated in the form of a law;
- the criterion should be heuristic, that is, it should allow to accumulate experience, develop intuition;
- the criterion should characterize the best and worst systems within its limits.

The purpose and tasks of the study. The goal is to determine a generalized criterion that would allow for optimal image processing in terms of the most important performance indicators. It is necessary to define a specific algorithm of criteria for evaluating efficiency, quality and optimization.

Using this algorithm, we can solve different problems. There are: building mathematical model of image processing system, building model of system functioning, development and optimization of the image processing system functioning algorithm, system synthesis and analysis, selection of system implementation elements, alignment of systems elements between each other as well as evaluation of image processing system efficiency, quality and optimization.

Selection and justification of private criteria. The first stage of solving the problem is the selection of a set of private criteria that characterize the system quite accurately. Let’s consider the image processing system as an object. The method allows you to highlight the image on a halftone object in real time [7].

We can identify the following private criteria that characterize this specific system:

1. system speed or time consumption;
2. system functionality;
3. information capacity of the system;
4. accuracy of system operation;
5. the probability of system task performance;
6. cost of system implementing;
7. hardware costs used for system implementation;

Let's consider the necessity, possibility and sufficiency of using each of the above criteria to assess the quality and optimality of this system.

1. Speed is the time taken by the system to execute a given image processing algorithm. The type of mathematical model of this criterion will depend on the specific type of system. When implementing the algorithm with a sequential system, the speed of operation is calculated in the following way:
\[ T(x, y, z, t) = \sum_{k=1}^{L} \sum_{j=1}^{M} \sum_{i=1}^{N} T(x_i, y_j, z_k, t) + \sum_{k=1}^{L} T(z_k, t), \]

where \( T(x_i, y_j, z_k, t) \) - preprocessing time;
\( T(z_k, t) \) - feature selection time;
\( M \) and \( N \) - the number of rows and columns of the image;
\( L \) - the number of discretization levels.

When implementing the algorithm with a parallel system, the speed will be calculated as:

\[ T(x, y, z, t) = \sum_{k=1}^{L} T(z_k, t) + T_0, \]

where \( T_0 \) - preprocessing time.

2. The functionality of this method should have:
   a) the possibility of work with halftone images of different brightness gradations;
   b) invariance of the method to affine transformations (rotation, scale change).

3. The criterion of the information capacity of the system [8, 9] can be considered as the ability of the system to issue the maximum amount of received information about the image of the object after its processing:

\[ \text{Imax}(x, y, t) = \sum_{i=1}^{N} \sum_{j=1}^{M} \text{Imax}_{i,j}(t), \]

where \( \text{Imax}_{i,j} \) - the maximum amount of information for \( i, j \) image pixels.

4. The accuracy of the system can be considered as: 1) the accuracy of the method implemented by the system and 2) as the accuracy of the system equipment. The accuracy of the method is most characterized by the probability of an erroneous decision [9]. A similar problem is considered in [10] which consists in the fact that it’s necessary to determine whether the received signal at each pixel of the image is useful or noise. The following errors may occur: 1) the decision is made that signal is useful when it’s noisy; 2) the signal is useful, but it’s decided that it belongs to noise. The first error is an error of the first kind or "false alarm". The second is an error of the second kind or "missing signal". The amount of errors of the first and second kind is estimated by the conditional probabilities \( \alpha \) and \( \beta \) of false decisions about the presence of a useful signal when in reality it is absent and about the absence of a signal when it really is. The probability of a wrong decision on one pixel is:

\[ P_{i,j} = q_{i,j} \alpha_{i,j} + p_{i,j} \beta_{i,j}, \]

where \( q_{i,j} \) and \( p_{i,j} \) - a priority probabilities of the absence and presence of a useful signal on \( i, j \) pixels.

The total probability of a false decision for the entire image will be:

\[ P_{er} = \sum_{i=1}^{N} \sum_{j=1}^{M} q_{i,j} \alpha_{i,j} + p_{i,j} \beta_{i,j} \]

Another criterion that reflects accuracy is the probability of recovery.

A lot of attention was paid to the construction and development of a system of quantitative measures of the accuracy of monochrome images restoration. It is indicated in [11] that adequate accuracy measures should be consistent with the results of subjective evaluations for a wide class of images, without requiring super complicated calculations. In addition, it is desirable for these measures to have
a simple analytical form – then they could be applied as optimality criteria in the optimization or selection of image processing system parameters. The specified requirements are fully consistent with the general requirements for the criteria formulated earlier. Quantitative measures of accuracy of monochrome images restoration can be divided into two groups: single and paired. A single measure is a number that is compared to an image based on an analysis of its structure. A paired measure is a numerical result of a mutual comparison of two images, for example, a reference and a real one.

As a criterion of efficiency and optimality of the method we can take the root mean square error of correction [12].

Let’s say \( U = \{F(i, j)\} \) - ideal picture, and \( i=1, \ldots, M, j=1, \ldots, N \) - coordinates of image elements, \( M \) and \( N \) - the number of line elements and lines in the frame. The image we observe \( V = \{\tilde{F}(x, y)\} \), which was formed as a result of system processing according to a given algorithm of some initial image. The quality of the algorithm will be better in accordance with the match of ideal and resulting image, therefore it is necessary to determine the "closeness of the images".

It is convenient to start with the quadratic measure of closeness - the Euclidean distance [1, 11]. The square of the Euclidean distance between the images is the square of their difference:

\[
\|\Delta\|^2 = \sum_{i=1}^{N} \sum_{j=1}^{M} \Delta^2(x, y),
\] (1)

where the difference image is

\[
\Delta(x, y) = V(x, y) - U(x, y)
\] (2)

To obtain a measure of the corrective transformation quality for some set of initial images as a whole, which would not depend on the implementation of the disturbance, it is convenient to take the mathematical expectation of the value \( \|\Delta\|^2 \) (1). By averaging it both over the set of initial images and over different realizations of the obstacle, we obtain the following value:

\[
\varepsilon^2 = \left\langle \|\Delta\|^2 \right\rangle = \sum_{i=1}^{N} \sum_{j=1}^{M} \varepsilon^2(x, y)
\] (3)

where \( \varepsilon^2(x, y) = \left\langle (V(x, y) - U(x, y))^2 \right\rangle \), is called root mean square correction error (RMSE).

It can be used as a criterion for the optimality of the correction. The best image is considered to be the one for which the RMSE is minimal. However, such a criterion does not meet the requirement of variability within defined limits, as there are difficulties in establishing its upper limit.

One of the reliability criteria is the normalized root mean square error (NRMSE) [12, 13].

\[
NRMSE = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} (v\{F(i, j)\} - v\{\tilde{F}(i, j)\})^2}{\sum_{i=1}^{N} \sum_{j=1}^{M} v\{F(i, j)\}^2},
\] (4)

where \( v\{\cdot\} \) - a certain (possibly non-linear) operator;

\( F(i, j) \) - ideal discrete image, \( \tilde{F}(i, j) \) - the image under study.
The most common fidelity criteria, which are based on estimates of normalized root mean square errors (NRMSE) for discrete images are listed in Table 1.

**Table 1. Accuracy criteria based on NRMSE for discrete images**

<table>
<thead>
<tr>
<th>Case</th>
<th>Formula</th>
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| 1. Without transformation | \[
NRMSE = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} [F(i, j) - \bar{F}(i, j)]^2}{\sum_{i=1}^{N} \sum_{j=1}^{M} [F(i, j)]^2}
\]
| 2. With transformation by decomposition elements | \[
NRMSE = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} [G(i, j) - \bar{G}(i, j)]^2}{\sum_{i=1}^{N} \sum_{j=1}^{M} [G(i, j)]^2}
\]  
\[G(i, j) = [F(i, j)]^w\]
| 3. With the Laplace transform | \[
NRMSE = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} [G(i, j) - \bar{G}(i, j)]^2}{\sum_{i=1}^{N} \sum_{j=1}^{M} [G(i, j)]^2}
\]
\[G(i, j) = F(i + 1, j) + F(i - 1, j) + F(i, j + 1) + F(i, j - 1) - 4F(i, j)\]
| 4. With the use of convolution | \[
NRMSE = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} [G(i, j) - \bar{G}(i, j)]^2}{\sum_{i=1}^{N} \sum_{j=1}^{M} [G(i, j)]^2}
\]
\[\text{where } G(i, j) = [F(x, y) \cdot H(x, y)]^w (x - i\Delta x, y - j\Delta y)\]

Another common accuracy criterion is the peak root mean square error (PRMSE), which is determined by the ratio

\[
PRMSE = \frac{1}{N \cdot M} \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} [G(i, j) - \bar{G}(i, j)]^2}{A^2},
\]

where \(G(i, j)\) - a transformed image that meets the definition given in Table 1, and number \(A\) is equal to the maximum value \(G(i, j)\).

It is also convenient to consider the root-mean-square error as a signal-to-noise ratio:
Selection of a generalized criterion. To form a generalized criterion, it is necessary to choose such a private criteria that characterize not only the image processing system, but also the way implemented by this system.

Performance, cost of the system, hardware costs characterize the system itself and depend on its specific type, only indirectly on the method. Evaluation according to the "functional capabilities" criterion is complicated due to the complexity of the mathematical formalization of this criterion. Therefore, the criteria that characterize the quality of the method are the information capacity, the probability of completing the task, and the accuracy of the image processing algorithm.

To derive a generalized criterion, let’s consider the concept of a real and ideal system [14]. A real system is understood as a system that processes and outputs information according to a given algorithm, chosen under the condition of obtaining the maximum amount of information about the image.

A potential or ideal information system is understood as a system that works in the same way as a real one, what is more, it completely removes uncertainty from the object and the system itself. Based on that, the generalized criterion can be represented as [14]:

\[
E(t, \tau) = \frac{E_r(t, \tau)}{E_p(t, \tau)},
\]

where \(E_r(t, \tau)\) - efficiency of the real system;
\(E_p(t, \tau)\) - efficiency of the potential (ideal) system.

Effectiveness of real and potential systems, can be defined as:

\[
E_p(t, \tau) = \frac{B_{\text{max}}}{C_{\text{min}}},
\]

where \(B_{\text{max}}\) - maximum benefit;
\(C_{\text{min}}\) - minimal costs.

\[
E_r(t, \tau) = \frac{B_r}{C_{\Sigma}},
\]

where \(B_r\) - real benefit;
\(C_{\Sigma}\) - total costs.

The benefit can be based on any of the previously discussed criteria. Thus, the efficiency of the system according to the information criterion, in relation to the system under consideration, can be estimated by the following expressions:

\[
E_t = \frac{I_t(x, y, t)}{C_{\Sigma}} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} I_{r, i, j}}{C_{\Sigma}}
\]
The defined complexity is the derivation of a generalized criterion, if the accuracy of restoration is considered as its argument. It is difficult to determine the efficiency separately for real and ideal systems, for example, relating the normalized root mean square error to costs, as it includes the concept of parameters of ideal and real systems [15]. However, NRMSE has the highest priority for the evaluation of the effectiveness of this type of system. The advantages of this criterion are listed below:

- The criterion reflects objective reality and characterizes efficiency.
- The criterion has a physical interpretation. This is the normalized root mean square error of image filtering and restoration.
- The criterion is statistical, since its arguments are random values or their probabilistic characteristics.
- The criterion is not difficult to calculate and can be calculated using appropriate programs.
- The criterion has defined limits $E_{\text{NRMSE}} \leq \frac{\max \text{NRMSE} - \min \text{NRMSE}}{E}$, where $E_{\text{NRMSE}}$ - specific limit value calculated for a specific type of images for which optimization is performed. At the same time, the more effective the algorithm, the smaller the error, and the closer $E_{\text{NRMSE}}$ to zero.
- The criterion is normalized.

The main disadvantage of such a criterion is that it doesn’t take into account equipment costs. To evaluate according to the cost criterion, it is necessary to know the costs of development, production and operation of the system, as well as the costs associated with the fulfillment of speed requirements. Generalized total costs can be determined by the expression:

$$C_\Sigma(t) = C_D(t) + C_M(t) + C_E(t) + \Delta C_T(t)$$  \hspace{1cm} (12)$$

where $C_D, C_M, C_E$ - the cost of development, production, and operation of the system; $\Delta C_T$ - additional costs to obtain the required speed.

$$\Delta C_T = K_T T$$  \hspace{1cm} (13)$$

where $K_T$ - specific coefficient that solves the dimensionality problem.

The minimum costs are:

$$C_{\min}(t) = C_D(t) + C_M(t) + C_E(t) + \Delta C_T(t)$$  \hspace{1cm} (14)$$

According to the cost criterion efficiency can be presented:

$$E_C = \frac{C_{\min}}{C_\Sigma}$$  \hspace{1cm} (15)$$
The criterion varies within:

$$0 \leq E_C \leq 1,$$

At the same time, when $E_c$ is close to 0, the system is inefficient, when $E_c$ is close to 1 – the system is potentially efficient.

The main disadvantage of the cost criterion is its one-sidedness, it means that only costs are taken into account.

Based on the above criteria and their properties, it is possible to formulate the main requirement of system optimality: the optimal system is the one where the normalized root mean square error is minimal at the same costs and given speed.

However, the following requirement will be more convenient: the optimal system is the one for which the cost criterion is maximal at a given NRMSE and a given speed.

Then the cost criterion will be represented by the formula:

$$E_C = \frac{C_D(t) + C_M(t) + C_E(t)}{C_D(t) + C_M(t) + C_E(t) + \Delta C_T(t) + \Delta C_{NRMSE}(t)}$$  \hspace{1cm} (16)

where $\Delta C_{NRMSE}(t)$ - the cost of additional costs to achieve the required accuracy.

$$\Delta C_{NRMSE} = K_{NRMSE} \cdot NRMSE,$$

where $K_{NRMSE}$ - the coefficient that solves the dimension problem.

If talk about efficiency $E_c$ in this problem [16], we mean some functionality of the speed of processing of requests $V_g$ and processing reliability, which is expressed in terms of errors of the first and second kind with the corresponding probabilities $P_I$ and $P_{II}$ [9,12]:

$$E = \psi(V_g, P_I, P_{II})$$

The cost is calculated by the formula:

$$C = a_1 V_{inf} + a_0.$$

1. With restrictions on the duration of the system channel with guaranteed quality of task performance: $t_d < T_{max}$, when $t_d > T_{max}$ $C = a_2 \cdot V_{inf}$, where $a_1$ and $a_2$ - weighting factors selected experimentally, based on the results of analysis of the system processing state.

If $a_i > a_0$, where the coefficient $a_i$ is determined by the method of expert assessments based on the results of statistical data processing, then, in this case, a linear dependence is proposed as a first approximation (Fig. 1).

The task of choosing a breaking point is subjective, so let's consider another cost function: $C = a_{12} \cdot V_{inf} + a_0$. Coefficient $a_{12}$ is selected from the condition $a_1 \cdot V_{max} + a_0 = a_12 \cdot (V_{max})^2$ or $a_{12} = a_1 / V_{max}$. 

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Fig. 1. Dependence with guaranteed quality of processing

2. Alternative - the quality of the data processing is not guaranteed [16].

\[ C = a_1V_{\text{inf}} + a_{03} \]

Ensuring data processing is the task of the operator. In this case, we will also focus on the linear dependence as a first approximation, but with a different coefficient and breaking point (Fig. 2), and on the quadratic dependence, where \( a_{12} \) is chosen for the same reasons as \( a_{12} \).

Fig. 2. Dependence of the cost of the data processing mode in the system

Quadratic cost functions are widely used in the analysis and synthesis of large systems, including information and computing and control systems.

3. Force majeure (maximum computing resources), quality of processing is maximum, while cost is the second priority. It is necessary to ensure the guaranteed volume of the system channel \( V_{\text{chan}} \geq V_{\text{inf}} \).

When determining the required volume of the system channels, the volume of processed information is approximately 10-100 times less in a full-time situation than when an out-of-hours situation occurs. Then, the value increases according to the law \( C_{\text{max}} = (1 - e^{-a_{V_{\text{inf}}}}) \), and the graph looks in the next way (Fig. 3)
In some cases, the value of information is an absolute priority. Therefore, it is advisable to choose the information function of the species as a cost function: \( C = a_4 \cdot \log_2 (1 + V_{inf}), \quad V_{inf} > 0 \) (Fig. 4).

Thus, it is necessary to choose the appropriate cost functions proposed above to optimize the time of image acquisition and its processing in various situations [17-20].

**Criterion application algorithm for the image processing system.** For this system, there are \( X \) options that differ in their technical implementation and algorithm that they implement.

1. The cost of developing, manufacturing and operating the system is calculated \((C_D, C_M, C_E)\). Moreover, the cost of production includes hardware costs. It should be taken into account that for each of the \( X \) systems the parameter \( C_M \) will be different.

2. The speed \( T \) of each of \( X \) systems is calculated. We choose \( Y \) options from them that satisfy the condition \( T_Y \leq 20ms \), that is, they work in real time [21, 22]. Systems that do not meet this requirement should be refined or rejected.

3. \( L \) experiments are conducted for each of the \( Y \) selected systems. Next step – to determine the mathematical expectation for each element of the investigated systems. Knowing values of the signal on each element of the reference image, according to the formulas from the table 1 or by the formula (6) we can define NRMSE for each system. By setting defined values \( \text{NRMSE}_{\text{max}} \) we compare the obtained results. When performing the inequality \( \text{NRMSE}_{Y}^L \leq \text{NRMSE}_{\text{max}} \) a subset \( Z \) is formed.

4. For systems included in the subset \( Z \) the value criterion is calculated according to the formula (16). The closer the obtained value is to 1, the more efficient the system.

**Conclusions.** The process of control and management of complex informaciologic objects can be considered as the process of formation and regulation of physically independent or interconnected processes occurring simultaneously or with some time shift. In this case a technical system for control and management of image processing should be considered as a multidimensional complex system.
The obtained efficiency criterion is generalized, its arguments are the accuracy of system operation, speed of operation, and cost savings. The criterion is statistical, since all parameters are random values or their statistical characteristics. Relatively simple criterion is being calculated and it meets the requirement $0 \leq E \leq 1$. The closer the value is to one, the more efficient the system. Since the criterion includes indicators of accuracy and speed, it allows to accumulate experience as a result of application for systems with different accuracy and speed. The simulation results and obtained dependences of cost functions on the speed of information processing in the system are given.

Thus, the generalized cost criterion meets all the requirements for optimization and quality criteria, therefore it’s suitable for evaluating the effectiveness of image processing systems.

REFERENCES

Моделі критеріальної оцінки ефективності систем обробки зображень

В статті досліджуються критерії, які застосовуються для оцінки ефективності систем обробки зображень. Проаналізовані вимоги, які передбачаються для критеріїв ефективності. Проведено вибір і обгрунтування приватних критеріїв, які застосовуються для систем обробки зображень. Такі параметри як швидкодія, вартість системи, апаратурні витрати характеризують в більшій мірі саму систему і залежать від її конкретного виду. Показано, що критеріями, які характеризують якість методу обробки зображень, здатність виконання задачі, точність алгоритму оброблення зображень. Показано, що такий критерій як нормована середньоквадратична помилка найбільше відповідає вимогам до критеріїв ефективності. Досліджено моделі критеріїв ефективності, які основані на нормований середньоквадратичній помилці стосовно дискретних зображень. Проведені розрахунки моделі, вони відповідають вимогам до критеріїв ефективності, які основані на нормований середньоквадратичній помилці стосовно дискретних зображень. Одержано узагальнений критерій, аргументами якого є точність роботи системи, швидкодія, вартісні витрати. Запропоновано алгоритм застосування узагальненого критерію для системи обробки зображень.

Ключові слова: Критерій ефективності, система обробки зображень, нормована середньоквадратична помилка.