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## Theoretical foundations of calculation cylindrical parts tank car boilers using the MathCAD environment

*The article is devoted to the numerical calculation of the cylindrical part a tank car boiler, which is represented as a single-layer shell, and the study of its stress-strain state using the provisions of the semi-momentum theory shells. In some cases, under actual operating conditions of a tank car boiler, when the base metal wears out under the influence of corrosive phenomena arising from the interaction an aggressive environment and a storage and transportation tank, it becomes necessary to assess the stress-strain state and search for the most dangerous areas. The authors of this article propose a calculation algorithm that allows determining the stress state cylindrical shell of a structure, taking into account the decrease in metal thickness when corrosion occurs, using the method of calculated sections in the MathCAD software environment. The constructed mathematical model makes it possible to determine: the values of longitudinal and transverse displacements a flexible homogeneous shell of constant stiffness under the acting combined load; the values of normal forces, bending moments, and equivalent stresses in accordance with the adopted design scheme of the cylindrical part the boiler. The simplicity of implementing the proposed numerical algorithm makes it possible to use it in engineering practice, for example, during technical inspections and making decisions on the further safe operation tank car boilers according to the adjusted methodology at the early stages of research*

***Keywords:** boiler, cylindrical part, shell, method of design sections, equivalent stresses, algorithm.*

**Introduction.** When performing engineering calculations of the stress-strain state a cylindrical tank car shells, momentless shells are used as design models. In such calculations, the load in the form of internal pressure acting on the shell is considered constant in magnitude, and the stresses are determined without taking into account bending, torques, and transverse forces. At the same time, calculation errors are partially taken into account using various coefficients.

In fact, the design model of the cylindrical shell of a tank car boiler must take into account the uneven loading along the meridional coordinate, since the boiler perceives different inertial forces during the braking period at the initial and final moments of the change in the mode of movement of the car. In addition, the shell is loaded with variable pressure from the mass of the liquid cargo with a significant density along the circumferential coordinate. There is also a need to take into account the fastening of the tank shell to the frame: in most designs, a semi-rigid fastening of the circumference part with free support on the leg supports that take vertical and horizontal loads is used in the pivotal section, and in the places where the shaped feet are located, a rigid fastening is used to prevent the boiler from moving in the longitudinal direction under the action of traction and braking forces.

The determination of the stress-strain state requires the use of approximate calculation methods, of which the most promising is the numerical method using the semi-momentum theory of shells.

**Analysis of recent research and problem statement.** The use and development of the semi-momentumless theory of shells as a mathematical basis for analyzing the stress-strain state of tank railcars remains one of the most important problems in solid mechanics today. A rather difficult task is to develop an algorithm and create an optimal mathematical model for determining the stress-strain state of a cylindrical boiler shell resting on the supporting parts of the car's load-bearing structures, taking into account its geometric features and contact perception and transmission of loads. The development of modern approaches to applying the provisions of the semi-instantaneous theory of shells is based on the general theory created by O. Cauchy, S. Poisson, A. Saint-Venant, G. Kirchhoff, A. Lyav, V. Vlasov, V. Darevsky, and other scientists. In particular, O. Cauchy proposed the method of step series when considering static and dynamic loads of flat and curved plates behind a cylindrical surface, and G. Kirchhoff became the founder of the hypothesis of a straight and undeformed normal, which was later used in the theory of shells by A. Lyav and V. Vlasov.

The problem of finding a sufficiently simple and accurate solution for determining the stress-strain state of a thin-layer shell under its loading by distributed and local forces in problems of an applied nature consists in various individual approaches to formulating the dependencies between stresses and strains on the basis of compliance with the Kirchhoff hypothesis or its rejection, as well as elasticity relations, which attracts the attention of specialists and scientists when assessing calculation errors.

In recent years, the study of the stress-strain state of cylindrical shells using the classical theory and its further development has been covered by scientists from different countries of the world. In particular, Schellhammer and Fries [1] proposed a revised theory of shells with the formulation of differential operators on a distributed surface in the global coordinate system, taking into account the tangential differential calculation. This eliminated the need to parameterize the geometry of the shell using the surface coordinates that were used previously. The authors of [2] presented the developed theory of deformable shells of the first order for the analysis of free and transient vibrations of layered open cylindrical shells with general boundary conditions. To analyze orthotropic cylindrical shells, researchers [3] derived a hierarchy of shell equations in the form of power series along the shell thickness. Depending on the linear-elastic state of the shell under the action of dynamic loads, the authors of [4] obtained systems of equations in integral-differential form instead of partial differential equations. In [5], a variant of the Kirchhoff-Lyav theory of shells extension was proposed for material anisotropy, not only in tension and out-of-plane bending, but also in in-plane bending by introducing an effective stress tensor, as well as in-plane and out-of-plane moment tensors, which are identified by the balance of mechanical forces.

**The purpose and tasks of the study.** Determining the effect of changes in the geometric dimensions of the metal shell of a tank car boiler on its structural strength remains an urgent scientific and technical problem that needs to be solved using refined methods and research programs both during operation and in determining the possibility of further extending the service life of railcars [6]. It is known that in accordance with the current regulatory documentation [7, 8], according to the established design load modes, the strength of individual parts of railcars is determined by the permissible stress values. The assessment of strength indicators can be carried out using the calculation method or during various types of field tests using measuring equipment. The need to assess the service life of a railroad car boiler structure requires finding the most rational methodology for studying its stress-strain state using adaptive mathematical calculation models.

The aim of the present study is to create a calculation algorithm that can determine the stressed state of the cylindrical shell of a tank car boiler, taking into account changes in the thickness of the metal of the structure due to corrosion phenomena [9] using the method of calculated sections in the MathCAD software environment.

**Materials and methods of research.** In the present work, the use of the classical semi-momentum theory of shells based on the Kirchhoff-Love hypotheses is proposed to determine the stress-strain state of the cylindrical part of a tank car boiler under the condition of reducing the shell thickness to obtain its maximum permissible values. It should be noted that in the presented calculation, the effect of static and dynamic loads on the boiler shell is taken into account. According to the design scheme (Fig. 1), the cylindrical shell is freely supported by absolutely rigid supports. This support implies that individual segments on the shell surface (Fig. 2) can move along the diaphragm in the direction of the cylindrical part, while in the final cross-sections of the shell, free rotation is allowed, but no movement is possible.

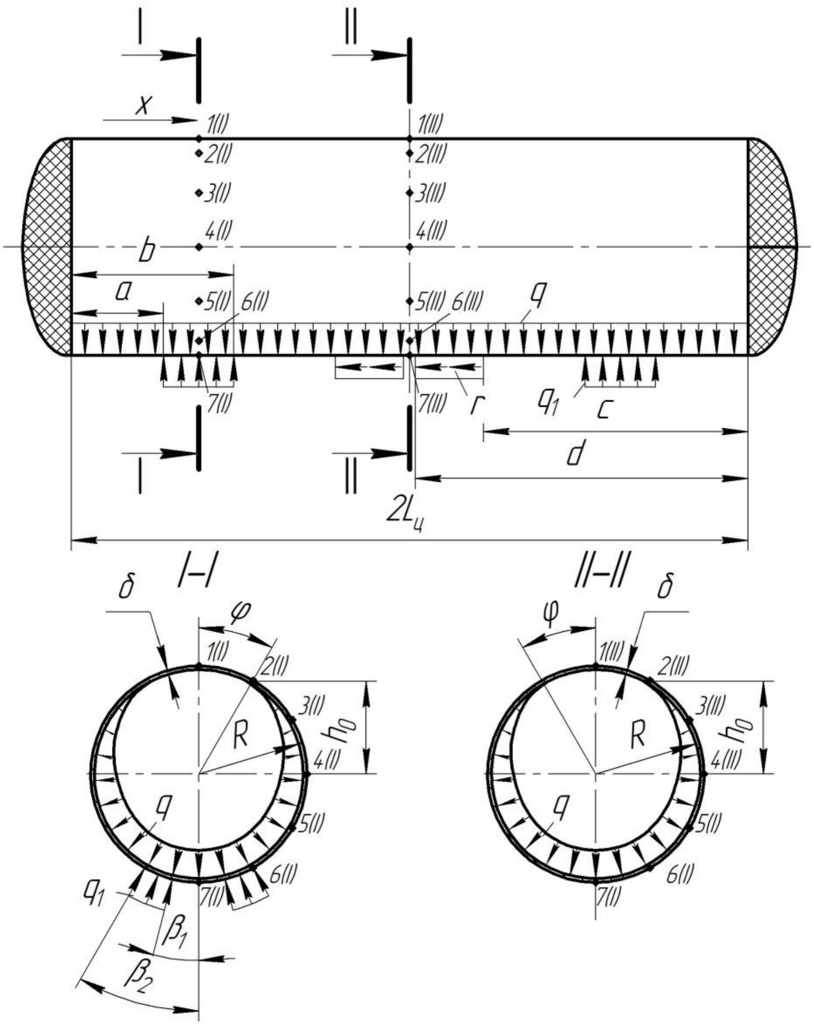

*Fig. 1.* **Design scheme of a cylindrical boiler shell**

In accordance with the adopted design scheme, the boiler of a tank car is subjected to the action of a distributed load from the mass of cargo $q$ with a height of $h_0$, which is perceived by the bed supports $q_1$, and the transfer of longitudinal forces $r$ to the shaped legs, which prevent the boiler from shifting during transient modes of movement, is taken into account. The geometric dimensions $b-a$ and $d-c$ determine the length of the supporting parts of the leg supports and shaped feet, respectively. The girth angles of the support parts for the kingpin part are determined by the difference $\beta_2 - \beta_1$.

In general, five internal forces act on the shell segment: normal forces $T$, shear forces $S$, transverse forces $N$, bending moments $M$, and torques $M_к$. In accordance with the semi-momentumless theory of shells, it is taken into account that bending moments and forces acting in the transverse direction, as well as torsional moments in the transverse and longitudinal directions, are equal to zero.

The algorithm for calculating the stressed state of the cylindrical part of the tank car boiler shell involves the following sequence of actions:

1) determination of the local reference coordinate system in accordance with the adopted design scheme;

2) setting the input parameters taking into account the geometry of the structure, acting loads, and elastic properties of the shell material;

3) determination of the bearing load in the considered design sections;

4) determination of the longitudinal load distributed over the contact surface of the boiler shell at the location of the shaped feet;

5) determination of vertical and longitudinal load coefficients using Fourier series [10];

6) forming a system of algebraic equations in matrix form to find the components of tangential displacement, shear forces, and normal forces;

7) determination of the coefficients of the series of tangential displacement, shear forces, and normal forces at the point of the shell by the angular coordinate in the studied section;

8) construction of bending moment diagrams and determination of their values in the section under consideration;

9) determination of equivalent stresses under the influence of force factors and comparison with permissible values;

10) variation of the input parameter of shell thickness change and repeated calculation cycle.

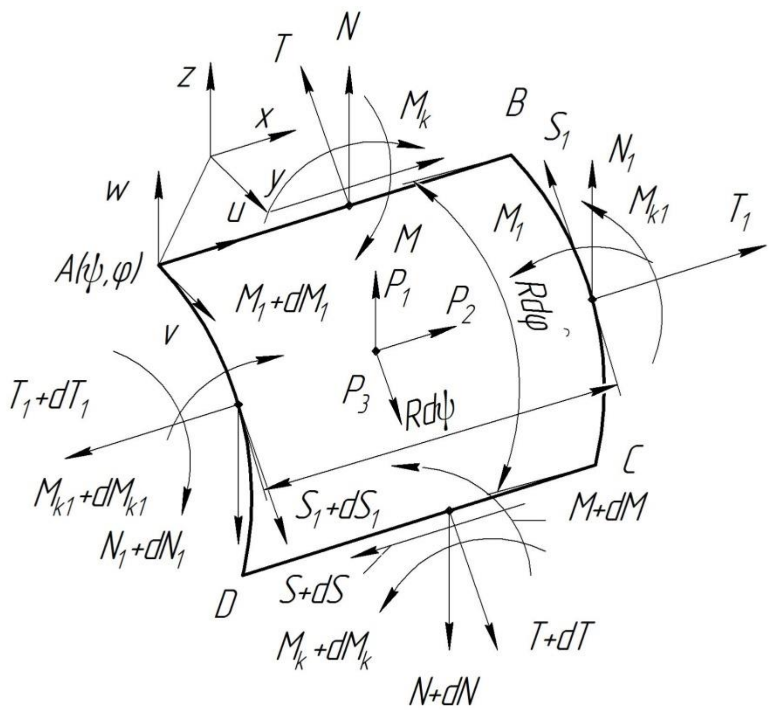


*Fig. 2.* **Effect of force factors on the boiler shell segment**

The method of calculated sections (sections I-I and II-II) Fig. 1 involves dividing the cylindrical part of the shell into a certain number of sections and determining the desired values (displacements, stresses) in the studied locations using the polar coordinate system. This makes it possible to determine the output parameters at each separately considered point, which is located on the surface of the cylindrical shell by the angular coordinate in accordance with the specified boundary interval of its length.

The system of differential equations for the equilibrium of the shell segment is as follows:

$$\left.\begin{aligned}&\frac{\partial T_1}{\partial \psi}+\frac{\partial S}{\partial \varphi}=0\\&\frac{\partial S}{\partial \psi}+\frac{\partial T}{\partial \varphi}-N=0\\&\frac{\partial^2 M_1}{\partial \psi^2}+\frac{\partial^2 M}{\partial \varphi^2}-2\frac{\partial^2 M_\kappa}{\partial \psi \partial \varphi}+RT=0\end{aligned}\right\} \tag{1}$$

where $T$, $T_1$ – normal forces;
$N$, $N_1$ – transverse forces;
$S$, $S_1$ – shear forces;
$M$, $M_1$ – bending moments;
$M_\kappa$ – torque;
$\psi$, $\varphi$ – coordinates of the cylindrical system;
$R$ – radius of the cylindrical shell of the boiler.

In general, linear deformations $\varepsilon_x$ and $\varepsilon_y$, angular $\gamma_{xy}$, the deformation of the curvature change $\chi_x$ and $\chi_y$, twisting $\gamma_{xy}$ expressed through displacement $w$, $v$, $u$. Let's write those that will be needed when deriving the main ones, which represent the complete system of equations of the semi-momentless theory:

$$\left.\begin{aligned}&\varepsilon_x=\frac{\partial u}{\partial x}\\&\varepsilon_y=\frac{\partial v}{R\partial \varphi}+\frac{w}{R}\\&\gamma_{xy}=\frac{\partial v}{\partial x}+\frac{\partial u}{R\partial \varphi}\\&\chi_y=\frac{1}{R}\left(\frac{\partial v}{\partial \varphi}+\frac{\partial^2 w}{\partial \varphi^2}\right)\end{aligned}\right\} \tag{2}$$

According to the theory, it is assumed that Poisson's ratio $\mu=0$, a linear and angular strain $\varepsilon_y=\gamma_{xy}=0$, that is, the elasticity ratio for forces $T_y$ and $S_y$ cannot be expressed through deformations. According to the considered option, such relations can be represented for force $T_x$ and the moment $M_y$ :

$$\left.\begin{aligned}&T_x=E\delta\varepsilon_x\\&M_y=\frac{E\delta^3}{12}\chi_y\end{aligned}\right\} \tag{3}$$

where $E$ – modulus of elasticity of the shell material;
$\delta$ – shell thickness.

Taking into account the effect of loads $P_1$, $P_2$ and $P_3$, distributed over the surface of the plane of the shell segment, we get the sum of the projections of the net forces on the normal, axis $x$ and tangential $y$ to the circumference of the cross-section, as well as the moment relative to the axis $x$. After the transformation, the differential equations of equilibrium of the shell segment will have the form (4).

The solution to the problem of determining additional internal forces in the cylindrical part of the boiler, caused by deformations of the contour of its cross sections, can be obtained on the basis of geometric equations (2), ratios (3) and equations (4). Let's convert these equations to a convenient solution, abandoning the traditional approach according to which the load $P_1$, $P_2$ and $P_3$ differentiable by any variable. Let's exclude the forces from the balance equations $N_y$, given that it is a moment ratio $\partial M_y$, which is applied at a distance $R\partial\varphi$ of the considered segment.

$$\left.\begin{aligned}&\frac{\partial N_y}{\partial\varphi}+T_y-P_1R=0\\&\frac{\partial T_x}{\partial x}R+\frac{\partial S_y}{\partial\varphi}+P_2R=0\\&\frac{\partial T_y}{\partial\varphi}+\frac{\partial S_y}{\partial x}R-N_y+P_3R=0\\&\frac{\partial M_y}{\partial\varphi}-N_yR=0\end{aligned}\right\}\tag{4}$$

After the transformations, we will have an equation of the form:

$$\left.\begin{aligned}&\frac{\partial^2 M_y}{R\partial\varphi}+T_y-P_1R=0\\&\frac{\partial T_x}{\partial x}R+\frac{\partial S_y}{\partial\varphi}+P_2R=0\\&\frac{\partial T_y}{\partial\varphi}+\frac{\partial S_y}{\partial x}R-\frac{\partial M_y}{R\partial\varphi}+P_3R=0\end{aligned}\right\}\tag{5}$$

Since for forces $T_y$ and $S_y$ there are no elasticity ratios, they can be found from equation (5). By turning off the power $T_x$ and moment $M_y$ let's transform these equations using relation (3) under the condition that the linear and angular deformations are equal to zero. Based on this, we will have:

$$\left.\begin{aligned}&w=-\frac{\partial v}{\partial\varphi}\\&\frac{\partial u}{\partial\varphi}=-\frac{R\partial v}{\partial x}\end{aligned}\right\}\tag{6}$$

In accordance with the first and fourth relations (2), expressions (3) and (6) yield:

$$\left.\begin{aligned} T_x &= -E\delta\int_{\varphi}\frac{\partial^2 v}{\partial x^2}Rd\varphi \\ M_y &= -\frac{E\delta^3}{12}\left(\frac{\partial^3 v}{\partial\varphi^3}+\frac{\partial v}{\partial\varphi}\right) \end{aligned}\right\} \tag{7}$$

By substituting expression (7) into the system of equations (5), we will have a system of integral differential equations of the form:

$$\left.\begin{aligned} &-\frac{E\delta^3}{12R^3}\left(\frac{\partial^5 v}{\partial\varphi^5}+\frac{\partial^3 v}{\partial\varphi^3}\right)+T_y-P_1R=0 \\ &E\delta\int_{\varphi}\frac{\partial^3 v}{\partial x^3}R^2 d\varphi-\frac{\partial S_y}{\partial\varphi}-P_2R=0 \\ &\frac{\partial T_y}{\partial\varphi}+\frac{\partial S_y}{\partial x}R+\frac{E\delta^3}{12R^3}\left(\frac{\partial^4 v}{\partial\varphi^4}+\frac{\partial^2 v}{\partial\varphi^2}\right)+P_3R=0 \end{aligned}\right\} \tag{8}$$

Such a system reflects the equilibrium conditions of an extremely small segment of the shell of the cylindrical part of the boiler, obtained taking into account geometric and physical relationships, i.e. it is equivalent to three groups of equations of the shell theory taking into account the hypotheses of the semi-momentless theory. The system of equations is complete and contains three equations with three unknowns.

The laws of load distribution acting on the boiler shell are represented by double trigonometric series with coefficients $P_{1mn}$, $P_{2mn}$ and $P_{3mn}$, determined by the Fourier formulas (9). The combination of trigonometric functions reflects the nature of the distribution of movements according to the corresponding load.

For the adopted settlement scheme at $x$, located within the length of the cylindrical part of the shell, there should be no movement $w$ and $v$, due to the fact that the contour of the cylindrical part of the boiler is not deformed at the end diaphragms. Sliding along the diaphragm can occur along the cylindrical part of the shell, namely $u \neq 0$.

$$\left.\begin{aligned} P_1 &= \sum_{m=1}^{\infty}\sum_{n=2}^{\infty}P_{1mn}\cos n\varphi\sin\lambda x \\ P_2 &= \sum_{m=1}^{\infty}\sum_{n=2}^{\infty}P_{2mn}\cos n\varphi\cos\lambda x \\ P_3 &= \sum_{m=1}^{\infty}\sum_{n=2}^{\infty}P_{3mn}\sin n\varphi\sin\lambda x \end{aligned}\right\} \tag{9}$$

where $m$ i $n$ – – numbers of row members;
$\lambda$ – ratio of the first number of the term of the series to the length of the cylindrical part of the shell.

The nature of the distribution of additional forces is reflected in contour deformations. When the load is symmetrical with respect to the longitudinal vertical plane of symmetry of the cylindrical part of the movement $w$ and $u$ symmetrical, and displacements $v$ directed obliquely. Therefore, they can be presented in the following records:

$$v = \sum\sum v_{mn}\sin n\varphi\sin\lambda x \tag{10}$$

$$\left.\begin{aligned} S_y &= \sum\sum S_{ymn} \sin n\varphi \cos \lambda x \\ T_y &= \sum\sum T_{ymn} \cos n\varphi \sin \lambda x \end{aligned}\right\} \tag{11}$$

In formulas (10) and (11) coefficients $v_{mn}$, $S_{ymn}$, $T_{ymn}$ unknown In the case of finding the values of the load coefficients $P_{1mn}$, $P_{2mn}$ and $P_{3mn}$ possible movements will look like:

$$\left.\begin{aligned} w &= 1 \cdot \cos n\varphi \sin \lambda x \\ v &= 1 \cdot \sin n\varphi \sin \lambda x \\ u &= 1 \cdot \cos n\varphi \cos \lambda x \end{aligned}\right\} \tag{12}$$

The equations of system (12) are continuous within the surface of the shell and correspond to the boundary conditions at the end sections of the cylindrical part of the shell and periodicity along the arc of the cross section.

Substituting the series from formulas (10), (11) and (12) into equation (8), the first of which is equivalent to the projection of internal and external forces on the direction of movement $w$, the second is equal to the projection on displacement $u$, and the third defines the projection on displacement $v$.

The work of force projections on the corresponding possible displacements is calculated by integration over the surface of the cylindrical part within the limits of $x=0$ to $x=2L_ц$, at the angular coordinate from $\varphi=0$ to $\varphi=2\pi$, that is:

$$\left.\begin{aligned} &\int_0^{2L_ц}\int_0^{2\pi}\left(\cos n\varphi \sin\frac{m\pi x}{2L_ц}\right)\left(\cos k\varphi \sin\frac{i\pi x}{2L_ц}\right)dxd\varphi \\ &\int_0^{2L_ц}\int_0^{2\pi}\left(\sin n\varphi \sin\frac{m\pi x}{2L_ц}\right)\left(\sin k\varphi \sin\frac{i\pi x}{2L_ц}\right)dxd\varphi \\ &\int_0^{2L_ц}\int_0^{2\pi}\left(\cos n\varphi \cos\frac{m\pi x}{2L_ц}\right)\left(\cos k\varphi \cos\frac{i\pi x}{2L_ц}\right)dxd\varphi \end{aligned}\right\} \tag{13}$$

Given the orthogonality of trigonometric functions, if $n=k$ and $m=i$ these integrals will be equal $L_ц\pi$, and in case $n\neq k$ and $m\neq i$ they will be zero.

The system of integral differential equations (8) in the process of determining the work will be reduced to the systems of algebraic equations (14) with respect to the coefficients of the series $v_{mn}$, $S_{ymn}$, $T_{ymn}$. According to these equations, the coefficients of the series are found, and according to formulas (11), there are intrinsically balanced forces, which are associated with deformations of the contour of the sections of the cylindrical shell. The forces at the studied points are the addition of series at the given coordinates $x$ and $\varphi$.

Coefficients of normal force and moment with known coefficients $v_{mn}$ can be determined by expression (7), then the system of equations will have the form (15).

$$\begin{vmatrix} -\dfrac{E\delta^3}{12R^3}n^3(n^2+1) & 0 & 1 \\ \dfrac{E\delta R^2\lambda^3}{n} & -n & 0 \\ \dfrac{E\delta^3}{12R^3}n^2(n^2-1) & \lambda R & n \end{vmatrix} \times \begin{vmatrix} v_{mn} \\ S_{ymn} \\ T_{ymn} \end{vmatrix} = \begin{vmatrix} P_{1mn}R \\ P_{2mn}R \\ P_{3mn}R \end{vmatrix} \tag{14}$$

$$\left. \begin{aligned} T_{xmn} &= -\frac{E\delta R\lambda^2}{n}v_{mn} \\ M_{ymn} &= -\frac{E\delta^3}{12R^3}n(n^2-1)v_{mn} \end{aligned} \right\} \tag{15}$$

For the coefficients of equations (15), we have:

$$\left. \begin{aligned} T_x &= \sum\sum T_{xmn}\cos n\varphi \sin\lambda x \\ M_y &= \sum\sum M_{ymn}\cos n\varphi \sin\lambda x \end{aligned} \right\} \tag{16}$$

Equivalent stresses are calculated according to the deformation energy criterion according to the formula:

$$\sigma_e = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2} \tag{17}$$

To ensure the required strength of the boiler shell, the condition must be met $\sigma_e \leq [\sigma]$, according to [7], and individual components of the equivalent stress (17) are determined by the system of equations of the form:

$$\left. \begin{aligned} \sigma_x &= \sigma + \frac{T_x}{\delta} \\ \sigma_y &= \frac{T_y}{\delta} \pm \frac{6M_y}{\delta^2} \\ \tau_{xy} &= \frac{S_y}{\delta} \end{aligned} \right\} \tag{18}$$

Hydrostatic pressure $q$ (Fig. 1) distributed according to the law:

$$q_1 = \gamma R(\cos\varphi - \cos\beta_0) \tag{19}$$

where $\gamma$ – specific weight of cargo;
$\beta_0$ – angle that determines the position of the free surface of the cargo.

When bearing loads are applied, the angular width of the planes is taken into account $\beta_2 - \beta_1$ in cities, leaning on horizontal supports, and the load is distributed $q$ oriented along the radius.

Load factors $P_{1mn}$, $P_{2mn}$ and $P_{3mn}$ for the supporting parts of the boiler without taking into account tangential loads will be determined by the formula:

$$\left.\begin{aligned} P_{1mn} &= \frac{4}{mn} q\left(\cos\lambda a - \cos\lambda b\right)\left(\sin n\beta_2 - \sin n\beta_1\right) \\ P_{2mn} &= \frac{4}{mn} r\left(\sin\lambda d - \sin\lambda c\right)\left(\sin n\beta_4 - \sin n\beta_3\right) \\ P_{3mn} &= 0 \end{aligned}\right\} \tag{20}$$

where $r$ – intensity of the distributed longitudinal load on the contact surface of the cylindrical shell and legs.

The longitudinal load distributed over the contact surface of the cylindrical shell and legs is determined by the formula:

$$r = \frac{T_1\left(m_к + m_в\right)}{4m_{бр}R\left(d-c\right)\left(\beta_4 - \beta_3\right)} \tag{21}$$

where $m_к$ – tank car boiler mass;

$m_в$ – mass of cargo transported in a tank car;

$m_{бр}$ – gross weight of the tank car;

$d-c$ – paw length;

$\beta_4 - \beta_3$ – angle of the boiler shell envelope with a shaped leg.

According to the presented methodology, the stress-strain state of the tank car boiler for improved sulfuric acid model 15-1548-02 was calculated. The calculation scheme of the cylindrical part of the boiler takes into account its geometric dimensions and operating loads (Table 1).

*Table 1.* **Input data for calculation**

| Parameter, dimension | Size |
|---|---|
| 1 | 2 |
| Tank car carrying capacity, kg | 67000 |
| Gross weight of the wagon, kg | 90300 |
| Wagon base, cm | 780 |
| Length of the cylindrical part of the boiler, cm | 927 |
| Radius of the cylindrical part of the boiler, cm | 110 |
| Inner angle of the girth of the supine support, degrees | 34 |
| Outer angle of the girth of the supine support, degrees | 36 |
| Inner angle of the girth of the shaped support, degrees | 5 |
| Outer angle of the girth of the shaped support, degrees | 9 |
| Distance from the extreme point of the cylindrical part of the boiler to the outer part of the lying support, cm | 50 |
| Distance from the extreme point of the cylindrical part of the boiler to the inner part of the lying support, cm | 150 |
| Distance from the extreme point of the cylindrical part of the boiler to the outer part of the shaped support, cm | 378,5 |
| Distance from the extreme point of the cylindrical part of the boiler to the inner part of the shaped support, cm | 448,5 |

*Table 1.* **Continued**

| | |
|---|---|
| Thickness of the boiler shell, cm | 0,7-1,1 |
| Longitudinal force, kg | 350000 |
| Modulus of elasticity of the material, kg/cm$^2$ | $2{,}1 \cdot 10^6$ |

Calculation of the strength of the tank car boiler is carried out according to the I regime, in which, according to the requirements [7] a longitudinal force of 3.5 MN acts on the car. The thickness of the boiler shell is a variable parameter that takes into account the amount of its change due to damage to the surface by corrosion [11]. At the same time, in the calculation, the thickness of the shell is assumed to be the same in all sections, that is, it is assumed that corrosion is evenly distributed over the entire surface of the metal at the same depth.

When determining the bending moments, the boiler is considered as a beam on two supports with a rigid contour of the cross section [12, 13]. The values of these moments depend on the longitudinal coordinate $x$, which determines the length of the cylindrical part of the boiler on the cross-section section in question, and the stresses in the cross-sections change in proportion to the bending moments.

Using MathCAD software [14-17] a plot of bending moments (Fig. 3) was constructed for individual sections of the cylindrical part of the boiler based on the equation written according to the syntax of the calculation program:

$$M(x) := \left| \begin{array}{l} q\dfrac{x^2}{2} \text{ if } \ 0 \le \mathrm{x} \le \mathrm{x}_L \\ q\dfrac{x^2}{2} - \dfrac{G}{2}\left(x - x_L\right) \text{ otherwise} \end{array} \right. \tag{22}$$

where $q$ – distributed load acting on the cylindrical shell of the boiler;
$x$ – coordinate of the half-length interval of the cylindrical part of the boiler;
$x_L$ – length of the cantilever part of the cylindrical part of the boiler, which is outside the base;
$G$ – vertical load from the weight of the cargo.

According to the established input data, the calculated maximum values of the bending moments of the shell at the supporting points of support at $x_1 = 73{,}5$ cm and in the middle part of the boiler at $x_2 = 463{,}5$ cm constitute $M(x_1) = 1{,}952 \cdot 10^5$ kg·cm and $M(x_2) = 5{,}301 \cdot 10^6$ kg·cm in accordance.

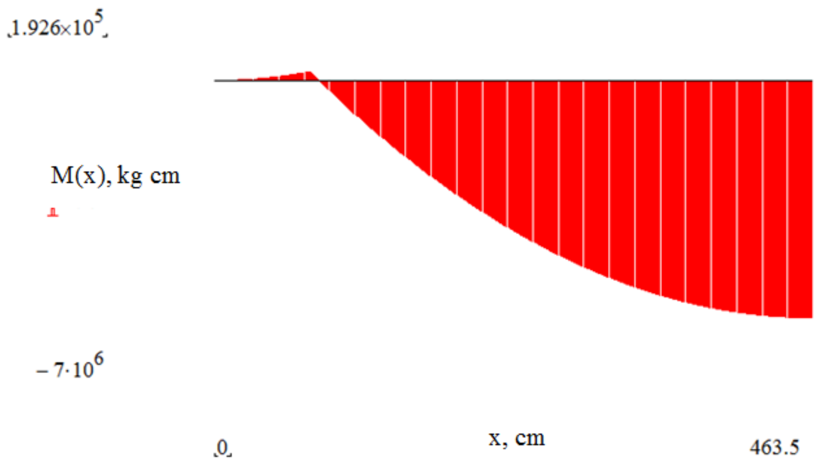


*Fig. 3.* **Bending moment diagram of the half-length of the cylindrical part of the boiler**

In accordance with the method of calculated intersections, to determine the forces and stresses acting on the boiler shell, the intersection point is first selected and the angular reference in the polar coordinate system is determined. Taking into account the fact that the support of the boiler on the lying supports and shaped legs is symmetrical in the profile plane, only half of the section is considered in the calculation (according to the following example, the left part is discarded, and the right part remains). Such an intersection has seven main positions, which correspond to a certain angle of inclination, in relation to the center, in steps of 30°. Positions are numbered clockwise. The angle that determines the coordinate, oriented along the cross-section of the cylindrical part of the boiler is $\varphi = 0...180°$. The number of intersections is chosen by the researcher depending on the task, and is determined by the direction coordinate $x$.

In this work, based on the described calculation algorithm based on the presented method, the pivot points of the boiler on the frame of the car and its middle part are considered as the main calculation sections. Taking into account the fact that the places of the greatest deformations and stresses occur in the contact zone of the boiler and supporting parts, the greatest attention is focused on the areas of positions 5, 6, 7 within the corner $\varphi = 120...180°$.

For cases of changes in the thickness of the boiler shell in the range from 1.1 to 0.7 cm in different design sections, according to the results of the calculations, it was established that the equivalent stresses (Fig. 4) exceed the permissible values of the yield strength of the material 09Г2С $[\sigma_m] = 325$ MPa at the minimum value of the shell thickness $\delta = 0.7$ cm in the pivot cross-section at the place where the boiler rests on the bearing supports and are 330.1 MPa (Fig. 5).

As can be seen (Fig. 4, a), for the kingpin section, the largest equivalent stresses in the boiler shell are in the range from 290° to 310° of the polar coordinate system, while their values at the accepted lower and upper limits of the shell thickness differ by a factor of 2.2. For the middle part of the boiler, the maximum values of equivalent stresses are in the range from 270° to 260° of the polar coordinate system and have a similar level of growth.

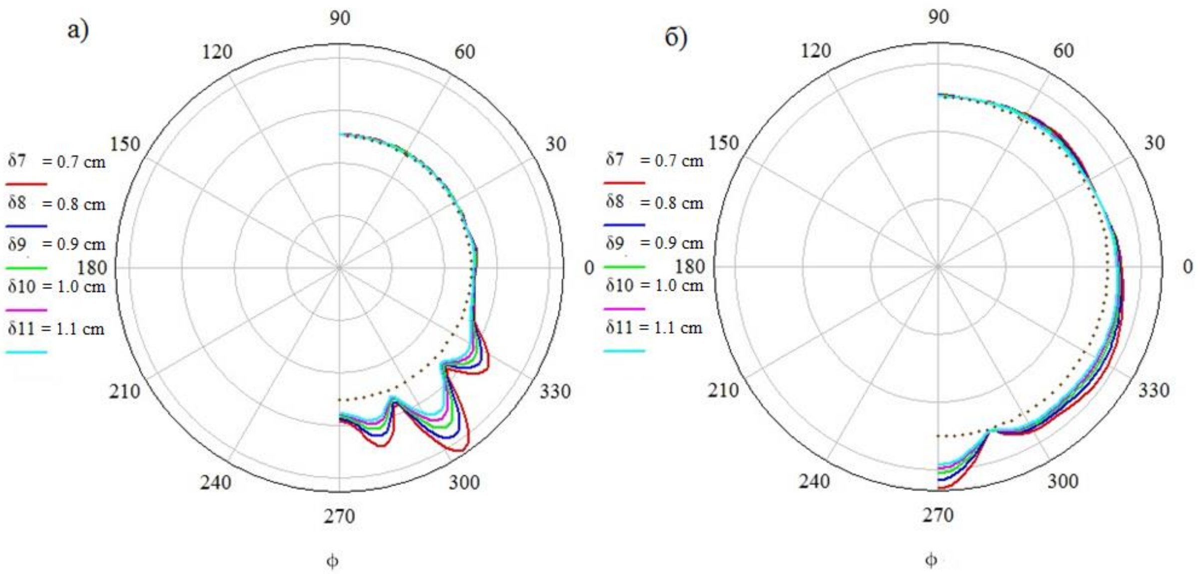


*Fig. 4.* **Diagram of the distribution of equivalent stresses in the tank car boiler shell in the design sections using MathCAD software:**
**a) pillar part; b) middle part of the boiler**

Determining the most stressed areas taking into account the reduction of the thickness of the metal shell is a priority task when calculating the strength of the tank car boiler, therefore changing the parameter $\delta$ is carried out from the permissible value to the minimum possible.

So for the case of the thickness of the boiler shell $\delta = 0.7$ cm it was established that the maximum values of tangential and radial displacements in the place of the supine support of the boiler on the frame are 1.95 mm and 10.54 mm, respectively. At the same time, the action of normal forces in the specified places corresponds to the values of 1.2 kN for the longitudinal direction and 1.76 kN for the transverse direction. For the cross-section in the middle part of the boiler, the maximum values of tangential displacements are 1.91 mm, and radial displacements are 6.13 mm. The maximum values of normal forces in this section for the longitudinal direction are equal to 0 kN, and for the transverse direction are 0.025 kN.

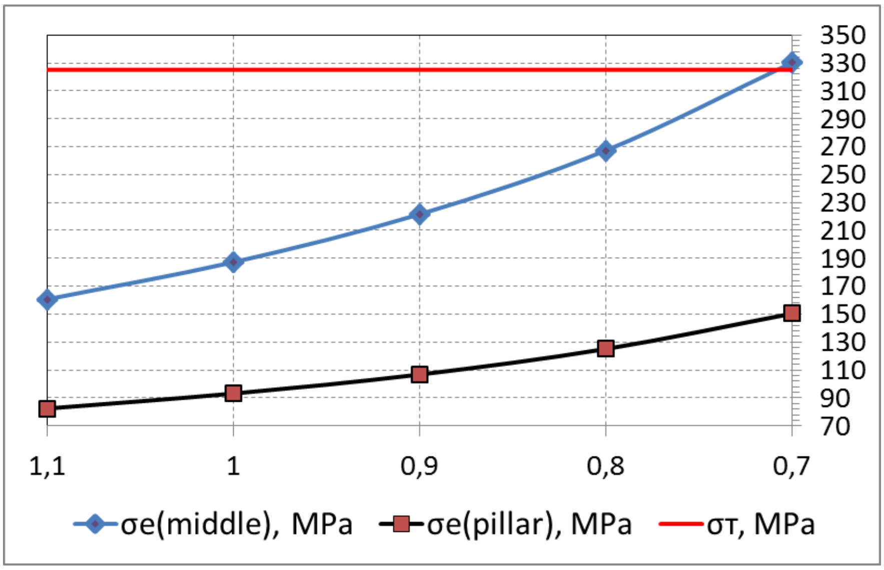


*Fig. 5.* **Diagram of maximum values of equivalent stresses (MPa) in the range of changes in the thickness of the boiler shell (cm)**

The obtained calculated values of stresses in the directions of action for the accepted minimum value of the boiler shell thickness are presented in Table 2.

*Table 2.* **Calculated values of the studied quantities for the case of the boiler shell thickness $\delta = 0.7$ cm**

| Parameter, dimension | Position number where the measurement is performed | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Angular position of the measurement point in the polar coordinate system, degrees | 90 | 60 | 30 | 0 | 330 | 300 | 270 |
| Normal stresses in the transverse direction in the cross section of the middle part of the boiler, MPa | 3,45 | 13,36 | 11,31 | 44,54 | 57,47 | 30,33 | 84,05 |
| Normal stresses in the longitudinal direction in the cross-section of the middle part of the boiler, MPa | 3,99 | 7,15 | 4,71 | 34,56 | 29,04 | 35,15 | 89,55 |
| Tangential shear stresses in the cross-section of the middle part of the boiler, MPa | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

*Table 2.* **Continued**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| Equivalent stresses in the cross-section of the middle part of the boiler, MPa | 6,45 | 18,04 | 14,26 | 40,48 | 76,25 | 56,76 | 150,4 |
| Normal stresses in the transverse direction in the pivot (support) section of the boiler, MPa | 5,46 | 2,96 | 1,92 | 2,81 | 51,56 | 106,1 | 92,91 |
| Normal stresses in the longitudinal direction in the pivot (support) section of the boiler, MPa | 2,71 | 2,41 | 0,82 | 18,68 | 115,1 | 202,8 | 32,35 |
| Tangential shear stresses in the pivot (support) section of the boiler, MPa | 0 | 2,88 | 3,71 | 3,51 | 11,61 | 15,96 | 0 |
| Equivalent stresses in the pivot (support) section of the boiler, MPa | 7,21 | 6,83 | 6,64 | 18,47 | 149,2 | 273,2 | 81,68 |

The distribution of equivalent stresses for each calculated value of the boiler shell thickness is presented in the form of a function that allows determining extremes localized within the angular position.

**Conclusions.** In the vast majority of works related to the calculation of cylindrical shells under local loads, equilibrium equations in real form are used as initial equations, which are reduced to the main 8th-order partial differential equation. The high order of this equation is a significant drawback when obtaining specific numerical values of the results. To eliminate the fundamental complications that arise when calculating the system of equilibrium equations, the determination of the vertical and longitudinal load coefficients is performed with the introduction of Fourier series.

Using the semi-momentum theory of shells, local problems were formulated and solved to determine the stress-strain state of a tank car boiler, taking into account the decrease in the thickness of the base metal of its cylindrical part when corrosion occurs. On the basis of these problems, analytical expressions for calculating the components of forces and stresses as functions of transverse and longitudinal coordinates were derived without using additional hypotheses about the nature of the stress distribution or displacements along the shell thickness.

As an example, the calculation of stresses in a cylindrical single-layer boiler shell under static load is presented. Explicit analytical expressions for all components of the stress tensor are obtained. MathCAD software was used to perform theoretical stress calculations. This made it possible to fully implement the calculation algorithm and automate the process of obtaining research results.

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## Теоретичні основи розрахунку циліндричних частин котлів вагонів-цистерн з використанням середовища MathCAD

*Стаття присвячена питанням чисельного розрахунку циліндричної частини котла вагон-цистерни, яка представлена у вигляді одношарової оболонки, дослідженню її напружено-деформованого стану з використанням положень напівбезмоментної теорії оболонок. У ряді випадків в дійсних умовах експлуатації котла вагон-цистерни при виникненні зношень основного металу під дією впливу корозійних явищ, що виникають під час взаємодії агресивного середовища та резервуара при зберіганні і транспортуванні, виникає необхідність оцінки напружено-деформованого стану та пошук найбільш небезпечних ділянок. Авторами даної статті запропоновано алгоритм розрахунку, за яким можна визначати напружений стан циліндричної оболонки конструкції з урахуванням зменшення товщини металу при появі корозії за методом розрахункових перетинів в програмному середовищі MathCAD. Побудована математична модель дозволяє визначати: величини поздовжніх і поперечних переміщень гнучкої однорідної оболонки постійної жорсткості при діючому комбінованому навантаженні; величини нормальних сил, згинальних моментів та еквівалентних напружень у відповідності до прийнятої розрахункової схеми циліндричної частини котла. Простота реалізації запропонованого чисельного алгоритму надає змогу використовувати його в інженерній практиці, наприклад, під час проведення технічних оглядів та прийнятті рішення щодо подальшої безпечної експлуатації котлів вагонів-цистерн за скоригованою методикою на ранніх етапах дослідження.*

***Ключові слова:*** *котел, циліндрична частина, оболонка, метод розрахункових перетинів, еквівалентні напруження, алгоритм.*