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# Analysis of minimum safe approach distances based on vessels navigation safety domain

*This article presents an analytical study of changes in the critical allowable approach distance between converging vessels, taking into account the shape of the vessel's safety zone. The research aims to address the important issue of ensuring maritime navigation safety by developing a mathematical approach for precise modeling of vessel domains under various approach scenarios. Analytical expressions are proposed and derived for calculating minimum safe distances for both elliptical zones and zones of complex configuration, allowing flexible assessment of approach situations depending on the relative motion of vessels. The analysis shows that although elliptical and complex-shaped domains differ geometrically, the nature of changes in critical approach distance in both cases remains similar, indicating the possibility of effective application of either model in practical conditions depending on the required level of detail and available computational resources. Graphical representation of the results clearly illustrates the dynamics of distance changes as a function of the angle between the courses of approaching vessels, which can be used in the development of software for navigation systems. The obtained dependencies allow not only quantitative assessment of allowable approach distances but also account for the influence of the approach aspect, which significantly affects the decision-making process by both navigators and automated collision avoidance systems. The results create a foundation for further improvement of collision avoidance algorithms and contribute to increasing the level of automation in navigation processes and overall maritime safety, especially in conditions of heavy traffic or restricted waterways.*

***Keywords:*** *safety domain, vessel collision avoidance, approach distance, elliptical domain, collision evasion, vessel traffic management.*

**Introduction**. In modern maritime transport, ensuring safe and accident-free navigation, particularly in restricted or narrow waterways, represents one of the most critical challenges facing navigational science and practice. The continuous increase in maritime traffic intensity, growing dimensions of modern commercial and passenger vessels, as well as the increasing complexity of navigational conditions in high-density traffic areas create unprecedented challenges for navigators and developers of vessel traffic management systems. Under such conditions, the issue of ensuring reliable collision avoidance between vessels on converging or crossing courses becomes paramount for minimizing collision risks.

Timely and well-founded decision-making regarding course or speed alterations requires precise criteria for evaluating safe distances between vessels. In this context, the concept of a vessel safety domain - a spatial area surrounding a vessel that should not be violated by other watercraft to avoid dangerous proximity - assumes particular significance. Accurate determination of the shape and dimensions of this zone constitutes a key element for developing effective algorithms for automated collision avoidance systems.

The relevance of this research is emphasized by the fact that in real maritime traffic conditions, vessels rarely approach on strictly opposing courses. Disregarding the approach direction, vessel type, maneuverability, speed, and dimensions inevitably leads to erroneous decisions. Therefore, the task of constructing an adaptive model of the vessel safety domain that accounts for variable parameters of the navigational environment and allows for precise determination of the critical allowable approach distance is not only scientifically interesting but also practically significant for enhancing overall maritime safety.

Despite significant contributions to the development of collision avoidance methods, a critical gap remains in understanding how vessel safety domains should dynamically adapt to changes in navigational conditions, vessel characteristics, and approach scenarios. Current models often employ static, symmetrical safety zones that do not account for the directional nature of collision risk and variable vessel maneuverability under different operational conditions. This research aims to address this gap by developing an adaptive model of vessel safety domains that incorporates multidimensional parameters, including vessel type, size, speed, maneuverability, traffic density, and environmental conditions.

Furthermore, the integration of such adaptive safety domains into automated collision avoidance systems presents significant challenges related to computational efficiency, real-time implementation, and compliance with international maritime regulations. This study proposes a novel framework that combines theoretical rigor with practical applicability in modern vessel traffic management systems, potentially revolutionizing the approach to collision avoidance in increasingly congested waterways.

**Analysis of recent research and problem statement.** The principles and methods of external control of vessel collision avoidance processes are detailed in works [1-3]. This fundamental research examines not only the primary methods of external control of vessel collision avoidance processes but also their theoretical foundation from the perspective of mathematical modeling and systems analysis. The proposed methods are based on utilizing domains of dangerous courses for approaching vessels and their dangerous speeds for comprehensive assessment of the risk level in approach situations. The authors developed an algorithmic procedure for determining optimal parameters of avoidance maneuvers considering navigational constraints and dynamic characteristics of vessels [4]. Special attention in the work is devoted to issues of coordinating vessel interaction during the execution of avoidance maneuvers and the possibility of accounting for the human factor in decision-making during critical situations. The proposed methodology also includes analytical expressions for evaluating the effectiveness of the selected avoidance maneuver in terms of minimizing loss of voyage time and fuel consumption [5, 6].

The method of vessel collision avoidance at sea by shifting to a line parallel to the path, at a specific angle to the programmed course line, is thoroughly investigated in the monograph [7]. The authors of the monograph presented not only the theoretical foundation of this method but also the results of its experimental verification based on simulation modeling and field trials. The proposed method allows for formalizing the process of selecting a safe maneuver considering hydrometeorological conditions, navigation intensity, and characteristics of the navigation area. As correctly noted in this fundamental work, a significant increase in the effectiveness of vessel collision prevention can be achieved through the development and implementation of innovative avoidance algorithms and intelligent decision support systems. The authors propose the concept of an adaptive collision prevention system capable of adjusting its parameters depending on changing external conditions and the behavior of other vessels [8-9].

Fundamental principles of external and locally-independent control of the collision avoidance process for vessels in dangerous proximity are comprehensively presented in work [10]. The authors thoroughly analyze the advantages and limitations of each control approach, as well as conditions for their effective application in various navigational situations. The work also contains a critical analysis of existing methods for vessel collision prevention and proposes ways to improve them considering modern technical capabilities and regulatory requirements. Special attention is devoted to issues of information support for the decision-making process and reducing uncertainty in assessing vessel movement parameters under conditions of limited visibility and complex navigational environments. The methodology proposed by the authors allows for formalizing the process of selecting an avoidance strategy taking into account the multi-criteria nature of this task and the uncertainty of initial information.

In cases of dangerous approach of a vessel with two targets, which significantly complicates the task of safe collision avoidance, work [11, 12] proposes an innovative method for forming domains of impermissible values of vessel movement parameters with respect to each target. The authors developed a comprehensive procedure for assessing the danger level of the emerging approach situation for each target using special collision risk indicators and a system for their integral evaluation. This research demonstrates the fundamental possibility of selecting an optimal avoidance maneuver through general evasion from both targets based on multi-criteria optimization. Three characteristic situations of a vessel approaching two targets are considered: crossing courses, overtaking, and head-on encounter. For each situation, algorithms for selecting a safe maneuver were developed and verified through simulation modeling methods.

The original description of the vessel collision avoidance process in terms of differential antagonistic game is presented in [13]. The authors formalize the vessel collision avoidance problem as a conflict situation with opposing interests among participants and propose methods for its resolution based on optimal control theory and differential games. This approach allows for consideration of active countermeasures between vessels and uncertainty in their intentions when selecting an avoidance strategy. The study defines necessary and sufficient conditions for the existence of a game solution, and develops effective numerical methods for determining it. The algorithms proposed by the authors can be implemented in onboard decision support systems and ensure guaranteed safe passage of vessels even under conditions of information deficit regarding the intentions of other traffic participants [14].

The work [15] examines in detail the emergency avoidance strategy in critical situations of excessive vessel proximity, when standard methods can no longer be applied due to deficits of time and maneuvering space. The authors propose an innovative approach to selecting parameters for emergency maneuvers, based on the principle of maximizing the vessel's dynamic capabilities while minimizing collision risk. The developed models and algorithms take into account actual vessel handling characteristics, including inertia, control system delay, and maneuvering intensity limitations. Methods for adapting the emergency avoidance strategy to specific navigation conditions and technical capabilities of the vessel are proposed. The results of theoretical research are confirmed by data from field tests and computer modeling.

The issue of selecting the optimal standard collision avoidance maneuver for two vessels is analyzed in detail in [16, 17]. The authors formulate the optimization problem as multi-criteria, taking into account not only the safety of the maneuver, but also its economic efficiency, operational constraints, and compliance with international regulations for preventing collisions at sea. The study develops mathematical models and optimization methods for parameters of standard collision avoidance maneuvers, including course alterations, speed adjustments, or their combination. Optimality criteria are proposed that account for safety, cost-effectiveness, and regulatory compliance. The results of theoretical research are validated through simulation data of typical vessel encounter situations under various navigational conditions [18].

In [19], innovative distributed algorithms for developing collision avoidance strategies involving multiple vessels in situations of dangerous proximity are presented. The authors propose a decentralized approach to coordinating vessel actions, based on information exchange regarding intentions and parameters of planned maneuvers through automatic identification systems. The developed algorithms ensure coordinated maneuvering of vessels without requiring centralized control, thereby enhancing the reliability and efficiency of the collision avoidance process in complex navigational environments. Paper [20, 21] presents the results of simulation modeling of distributed algorithms in various vessel traffic scenarios and demonstrates their advantages compared to traditional collision prevention methods.

Paper [22, 23] provides rigorous analytical expressions for calculating the boundaries of domains of dangerous courses and dangerous speeds under external control of the vessel collision avoidance process. The authors derived exact formulas for determining the boundaries of prohibited values of one vessel's courses and another vessel's speeds, taking into account their relative positions, dimensions of safety zones, and dynamic characteristics. Special attention is given to the graphical representation of these domains for visualization and operational assessment of the navigational situation. The proposed methods allow for real-time determination of safe vessel movement parameters and selection of optimal collision avoidance maneuvers. The results of theoretical research are confirmed by numerical modeling data and comparison with established empirical rules.

The theoretical foundation for an autonomous vessel collision avoidance system is presented in the fundamental work [24]. The authors developed a comprehensive collision avoidance algorithm based on principles of artificial intelligence and modern methods of optimal control. The study systematizes and generalizes various approaches to vessel control automation in collision prevention tasks. The authors distinguish two main research directions: the first is based on the application of mathematical models and analytical algorithms, while the second utilizes artificial intelligence methods, including neural networks, fuzzy logic, and evolutionary algorithms. A comparative analysis [25, 26] of the effectiveness of these approaches in various navigational situations is conducted, and recommendations for their practical application are formulated. The research also presents a concept for a hybrid collision prevention system that integrates the advantages of both approaches and ensures high reliability of decision-making under conditions of uncertainty in initial information and complex navigational environments. The architectural solutions and algorithms proposed by the authors can serve as a foundation for the development of advanced intelligent maritime safety systems.

**The purpose and tasks of the study**. Analysis of the current state of the problem indicates the necessity of employing more precise mathematical models and algorithms for determining safe vessel collision avoidance parameters, taking into account their maneuvering characteristics and specific navigational conditions. Traditional approaches to formalizing the vessel collision avoidance process are predominantly based on simplified safety domain models, which do not always adequately reflect actual navigational conditions.

As demonstrated in [27], the condition for safe vessel passage is determined by the equality of the closest point of approach distanceня $D_{min}$ the minimum permissible approach distance $D_d$, i.e. $D_{min}=D_d$ under the condition that vessels are converging. This fundamental principle serves as the starting point for developing effective algorithms for preventing vessel collisions at sea and forms the basis for mathematical modeling of the collision avoidance process.

Traditionally, the minimum permissible approach distance $D_d$ is independent of the relative positions of converging vessels and is assumed to be constant in magnitude, implying that a vessel's safety domain has a circular shape and is defined in relative motion space. This approach, although characterized by simplicity, fails to account for the maneuvering characteristics of different vessel types and the asymmetry of their safety domains under real navigational conditions.

However, in recent years, more than ten safety domain shapes distinct from circles have been proposed. Their application allows for more accurate modeling of vessel interaction dynamics and more efficient utilization of available navigational space. Therefore, the minimum permissible approach distance $D_d$ when using such domains is not constant but depends on the relative aspect of converging vessels, which significantly complicates algorithms for collision risk assessment and optimal avoidance maneuver selection.

The value of the minimum permissible approach distance between vessels, accounting for the safety domain shape, depends on the side of relative evasion and is determined by the limiting relative courses $K_{\mathrm{ot}}^{s}$ and $K_{\mathrm{ot}}^{p}$ and the bearing angles. Therefore, we denote $D_d^s$ and $D_d^p$ - as the minimum permissible approach distances for relative evasion to starboard and port respectively, as shown in Fig. 1.

Figure 1 presents a geometric scheme for determining the maximum permissible approach distance between vessels, taking into account the shape of the target's safe domain. The diagram shows two vessels: the own vessel and the target. Around the target, a safe domain in the form of an ellipse is shown, which defines the area within which approach is considered dangerous.

From the own vessel to the target, two directions are drawn, corresponding to possible evasion options - to the right and to the left. For each of these directions, the limiting relative evasion courses are determined: Kotp* - when deviating to the right, Kots* - when deviating to the left. Accordingly, the maximum permissible approach distances for each option are denoted as $D_{dp}$* (to the right) and $D_{ds}$* (to the left).

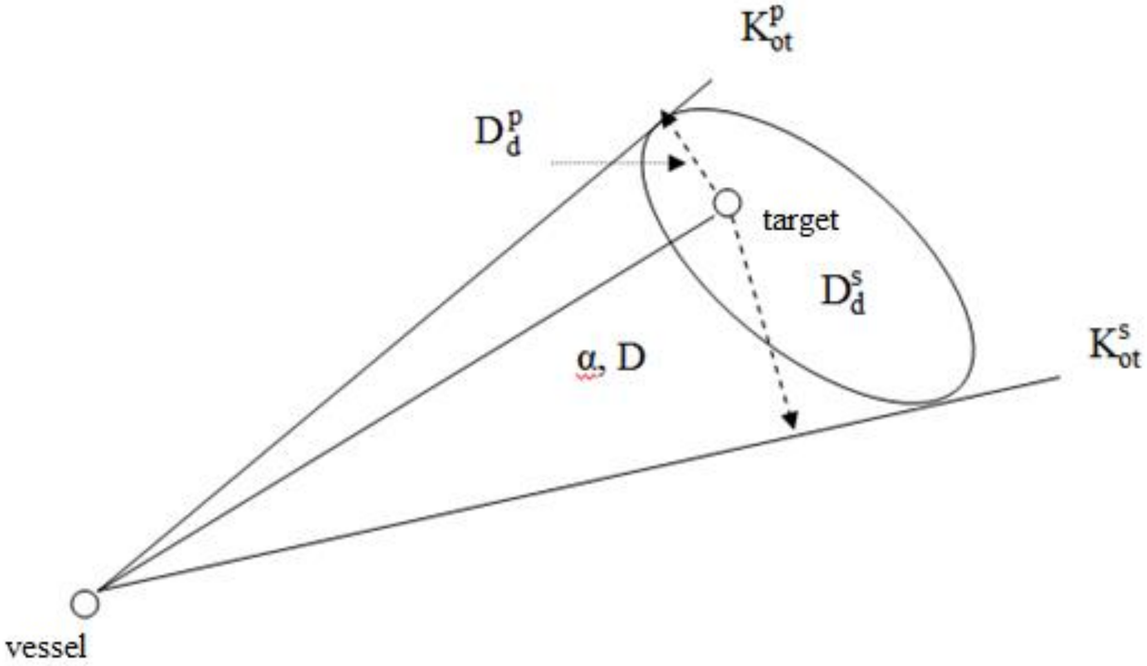

*Fig. 1.* **Minimum permissible approach distances**

The angle α denotes the course angle to the target, and D represents the current distance between the vessels. Dotted lines show the directions to the points of tangency of trajectories to the boundary of the safe domain, which determine the values of the maximum permissible approach distances for each evasion option.

Thus, the figure illustrates that the value of the maximum permissible approach distance depends both on the shape of the safe domain and on the chosen side of evasion, and is also determined by the corresponding limiting relative courses and course angles. This allows for obtaining expressions for calculating the maximum permissible approach distances in each specific case of maneuvering.

From this same figure, expressions for determining the minimum permissible approach distances can be derived

$$D_d^s = D\left|\sin(K_{\text{ot}}^s - \alpha)\right|, \quad D_d^p = D\left|\sin(K_{\text{ot}}^p - \alpha)\right|,$$

where $D$ and $\alpha$ are the distance to the target and bearing to the target, respectively.

To achieve the stated objective, it is necessary to solve several interrelated tasks. First, to derive analytical expressions for calculating the limiting relative evasion courses of a vessel for various safety domain shapes. Second, to develop algorithms for calculating the minimum permissible approach distance for safety domains of different shapes. Third, to investigate the dependency of the minimum permissible approach distance on target aspect for domains of elliptical and complex shapes. And finally, to conduct a comparative analysis of the characteristics of safety domains of various shapes and determine the most effective domain shape for practical application in decision support systems for vessel collision avoidance [28].

Let us consider two types of domains, for each of which it is necessary to find expressions for the limiting relative evasion courses $K_{\text{ot}}^s$ and $K_{\text{ot}}^p$ with subsequent development of procedures for calculating the minimum permissible approach distances $D_d^s$ and $D_d^p$. This task has significant practical importance, as it enables the development of more accurate algorithms for monitoring vessel traffic safety and optimizing their collision avoidance maneuvers.

First, let us examine the analytical expressions for limiting relative evasion courses for a domain of elliptical shape, which we will denote as $D_b^{(\text{El})}$. n this case, the relative course of minimum evasion $K_{\text{otymin}}$ is determined by the tangent to the domain boundar $D_b^{(\text{El})}$, he position of which depends on the target aspect. In [14], expressions for calculating the limiting relative evasion courses were derived, which are defined as follows:

$$K_{\text{ot}}^s = max\{\bar{K}_{\text{ymin1}}, \bar{K}_{\text{ymin2}}, \bar{K}_{\text{ymin3}}, \bar{K}_{\text{ymin4}}\}.$$
$$K_{\text{ot}}^p = min\{\bar{K}_{\text{ymin1}}, \bar{K}_{\text{ymin2}}, \bar{K}_{\text{ymin3}}, \bar{K}_{\text{ymin4}}\}.$$

In the last expression:

$$\begin{aligned} \bar{K}_{\text{ymin1,2}} &= arctg\frac{\bar{X}_o \pm b\sqrt{1-\frac{x_1^2}{a^2}}sin\,K_c + x_1\,cos\,K_c}{\bar{Y}_o \pm b\sqrt{1-\frac{x_1^2}{a^2}}cos\,K_c - x_1\,sin\,K_c}; \\ \bar{K}_{\text{ymin3,4}} &= arctg\frac{\bar{X}_o \pm b\sqrt{1-\frac{x_2^2}{a^2}}sin\,K_c + x_2\,cos\,K_c}{\bar{Y}_o \pm b\sqrt{1-\frac{x_2^2}{a^2}}cos\,K_c - x_2\,sin\,K_c}, \end{aligned} \tag{1}$$

where

$$x_1 = -\frac{a^2cb}{a^2+c^2r^2} + \sqrt{(\frac{a^2cb}{a^2+c^2r^2})^2 - \frac{a^2c^2(b^2-r^2)}{(a^2+c^2r^2)}};$$

$$x_2 = -\frac{a^2cb}{a^2+c^2r^2} - \sqrt{(\frac{a^2cb}{a^2+c^2r^2})^2 - \frac{a^2c^2(b^2-r^2)}{(a^2+c^2r^2)}};$$

$$c = \frac{a^2}{b(\bar{Y}_o \sin K_c - \bar{X}_o \cos K_c)};$$

$$r = (\bar{Y}_o \cos K_c + \bar{X}_o \sin K_c).$$

For calculating the limiting relative evasion courses and minimum permissible approach distances, a computer program was developed that, depending on the angle $\xi = K_{\text{ot}}^{s} - \alpha$ calculates the value of the minimum permissible approach distance $D_d(K_c)$, as shown in Fig. 2. This enabled obtaining quantitative characteristics of the dependency of the minimum permissible approach distance on the relative position of vessels and substantiating the selection of an optimal safety domain shape.

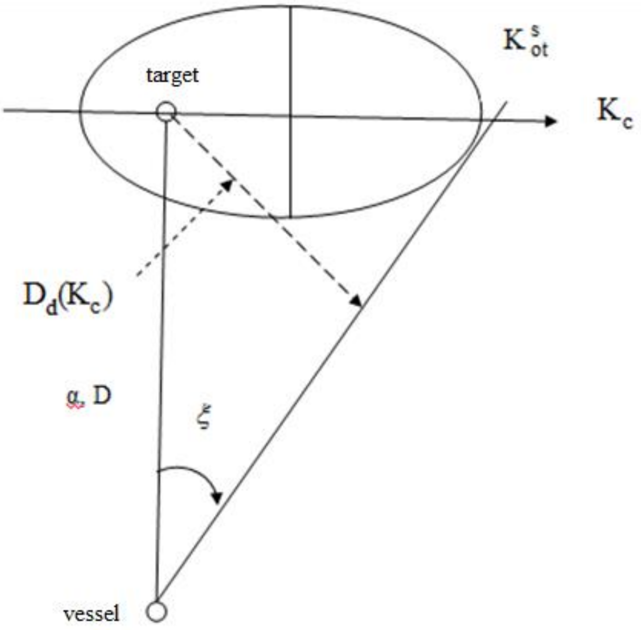

*Fig. 2.* **Dependence of distance $D_d(K_c)$ on angle $\xi$**

The figure schematically shows a geometric interpretation of the process of determining the limit distance of a vessel's approach to the target in the plane of relative courses. The oval in the upper part of the figure denotes the safe domain of the target, which is modeled as an ellipse. The center of the ellipse corresponds to the position of the target. Two lines are drawn from the target: one perpendicular to the major axis of the ellipse, the other at a certain angle to this axis, illustrating possible approach trajectories.

From the vessel, located in the lower part of the figure, two vectors are drawn:

- the first vector (solid line) connects the vessel with the target and indicates the current distance D between them;

- the second vector (dashed line) shows the direction to the point of tangency of the vessel's trajectory to the boundary of the target's safe domain, which determines the limit approach distance $D_d(K_c)$.

The angle ξ illustrates the deviation of the vessel's course relative to the target, and α is the angle between the direction to the target and the direction of the vessel's movement. The axis $K_c$ denotes the relative course of the vessel, and $K_{ot}^{s}$ is the evasion course.

Thus, the figure illustrates the spatial relationship between the vessel, the target, and the safe domain, as well as the geometric approach to determining the limit approach distance for constructing evasion algorithms.

By varying the bearing from від 5° to 360° he program calculated the corresponding values of the limiting relative evasion course $K_{\text{ot}}^{s}$ and distance $D_d(K_c)$. The calculation results are presented in Fig. 3, with the right part showing the graphical dependency of the minimum permissible approach distance on the value of the angle $\xi$. The obtained results allow us to conclude that there is a significant dependency of the minimum permissible approach distance on the target aspect, which must be taken into account when developing vessel collision avoidance algorithms.

Let us consider the calculation of limiting relative evasion courses for a safety domain $D_b^{(\text{RE})}$ complex shape, which represents a combination of half an ellipse and half a circle. As previously indicated, the limiting relative evasion course of a vessel is the tangent from the vessel's position to the target's safety domain. In [22], it is shown that in the general case, the relative minimum evasion course of a vessel $\bar{K}_{\text{ymin}}$:

$$\bar{K}_{\text{ymin}} = \begin{cases} K_{\text{otn}} + arcsin\dfrac{R_b}{D}, \text{ if } sin\,\beta < 0, \\ max\{\bar{K}_{\text{ymin1}}, \bar{K}_{\text{ymin2}}, \bar{K}_{\text{ymin3}}, \bar{K}_{\text{ymin4}}\}, \text{ if } sin\,\beta > 0, \end{cases}$$

where $\bar{K}_{\text{ymin1}}$, $\bar{K}_{\text{ymin2}}$, $\bar{K}_{\text{ymin3}}$ and $\bar{K}_{\text{ymin4}}$ are determined by expression (1),
$\beta$ - the bearing angle from the target to the vessel.

In Figure 3, the results of numerical modeling of changes in the maximum permissible approach distance between vessels depending on the target bearing angle with an elliptical shape of the safe domain are presented. The central part of the figure contains a polar diagram, where the elliptical safe domain of the target is highlighted in yellow, modeling the area within which the approach of vessels is considered dangerous. From the center of the domain, numerous radial lines are drawn in different directions, corresponding to different values of the bearing angle (from 5° to 360°). On each line, a point of tangency to the boundary of the ellipse is marked, which determines the maximum permissible approach distance for the corresponding direction.

To the right of the main diagram, a graph shows the relationship between the maximum permissible approach distance and the bearing angle value. The x-axis represents the angle (from 0° to 360°), and the y-axis shows the corresponding distance. The graph demonstrates the non-uniform nature of changes in this distance: maximum values are observed in directions that coincide with the major axis of the ellipse, while minimum values correspond to directions along the minor axis. This indicates a significant dependence of the maximum permissible approach distance on the target's aspect.

The obtained results confirm that when developing algorithms for vessel collision avoidance, it is necessary to consider not only the absolute value of the distance but also its dependence on the approach direction and the geometry of the safe domain. This approach allows for improving the accuracy and reliability of automated collision avoidance systems in complex navigational situations.

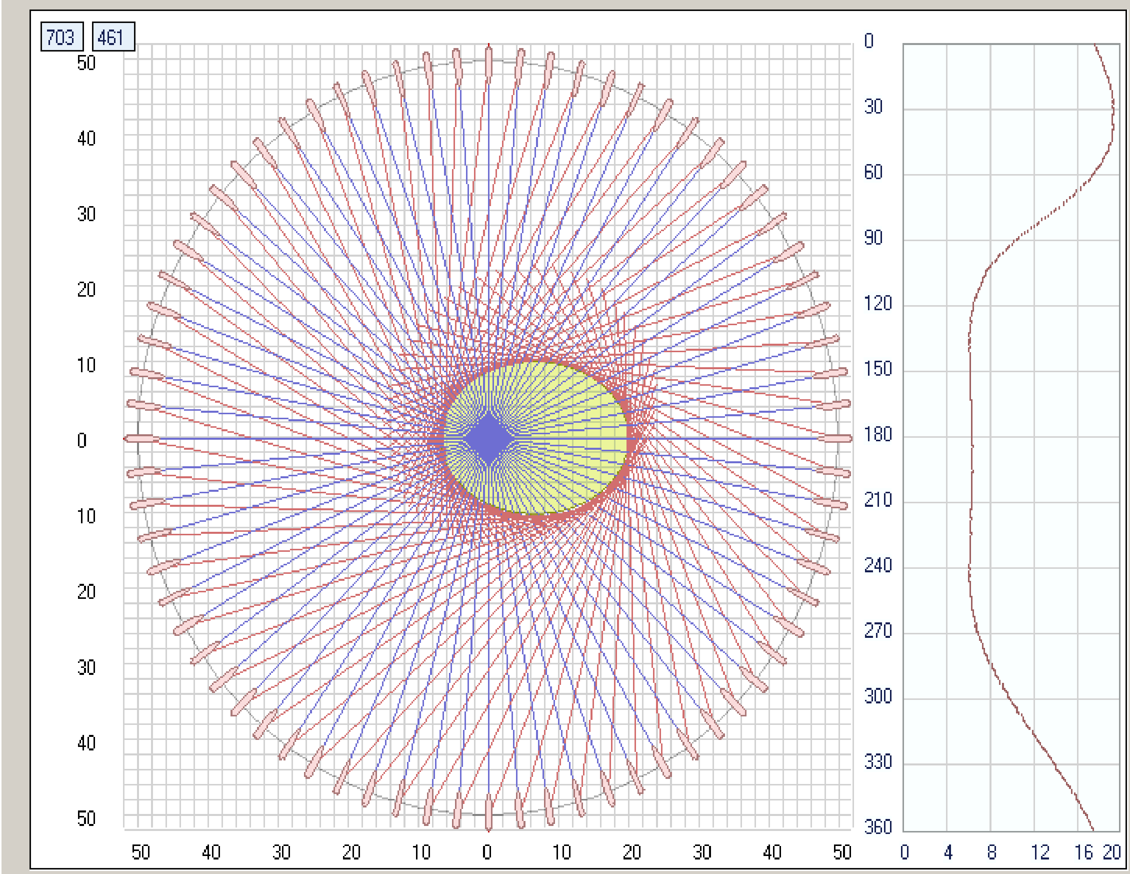

*Fig. 3.* **Change in distance $D_d(K_c)$ with elliptical domain shape**

As in the previous case, the developed computer program was used to calculate the limiting relative courses and values of the minimum permissible approach distance $D_d(K_c)$ for a domain $D_b^{(\mathrm{RE})}$ of complex shape, the nature of change of which depending on the angle is shown in Fig. 4. In the central part of the figure, a safe domain with a combined, asymmetric shape (green area) is shown, which combines the properties of an ellipse and a semicircle. This domain shape allows for more accurate consideration of the characteristics of vessels' mutual movement in various navigational situations.

From the center of the domain, numerous radial lines (red color) are drawn in all directions, corresponding to different values of the bearing angle from 0° to 360°. On each line, a point of tangency to the boundary of the complex domain is marked, which determines the maximum permissible approach distance for the corresponding direction. This allows for visual assessment of how the safe distance changes depending on the target's aspect.

To the right of the main diagram, a graph shows the relationship between the maximum permissible approach distance and the bearing angle. The x-axis represents the angle (from 0° to 360°), and the y-axis shows the corresponding distance. The graph demonstrates the non-uniform nature of changes in this distance: both maximum and minimum values are observed depending on the approach direction, reflecting the complexity of the domain geometry.

The results presented in Figure 4 allow for evaluating the influence of the safe domain shape on the maximum permissible approach distance and justifying the selection of the optimal domain shape for different types of vessels and navigational situations. This, in turn, contributes to improving the accuracy of collision risk assessment and the effectiveness of choosing the optimal evasion maneuver, which is important for the development of modern intelligent collision prevention systems, including those for autonomous navigation.

These results allow for evaluating the influence of the safe domain shape on the maximum permissible approach distance and justifying the selection of the optimal domain shape for different types of vessels and navigational situations.

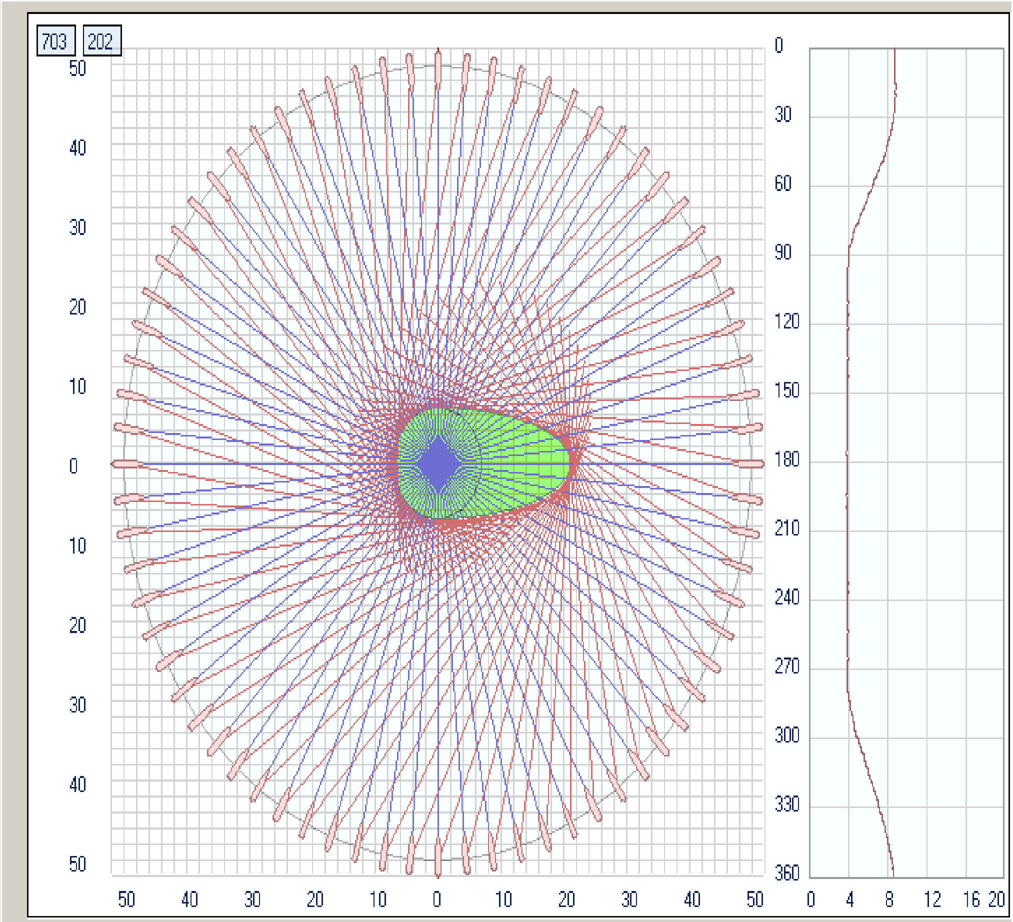

*Fig. 4.* **Change in distance$D_d(K_c)$ of a complex-shaped domain**

Analyzing the results presented in Figures 3 and 4, it can be noted that both safe domains-elliptical and complex shapes-demonstrate a similar pattern of changes in the maximum permissible approach distance depending on the vessel's aspect. In particular, for both domains, significant variability of this distance is observed depending on the target bearing angle: maximum distance values occur in directions close to the longitudinal axis of the vessel, while minimum values correspond to directions that match transverse aspects. This pattern indicates the necessity of considering the target's orientation when developing collision avoidance algorithms.

At the same time, the complex domain, which combines elements of an ellipse and a semicircle, provides a more flexible and realistic model of the safe zone, allowing for more accurate reproduction of vessel maneuvering characteristics in various navigational situations. However, given the similarity of the obtained results, for practical applications it is advisable to choose the domain that provides an optimal balance between assessment accuracy and computational efficiency. This is especially important for automatic collision prevention systems, where information processing speed and decision-making reliability are critical.

Below is a comparative table and graph illustrating the dependence of the minimum safe approach distance between vessels on the target bearing angle for two different forms of safety domain: elliptical (Fig. 3) and complex (Fig. 4). The data were obtained through numerical modeling and reflect the pattern of distance changes for different approach directions.

*Table 1*. **Comparison of Minimum Safe Approach Distances for Elliptical and Complex Safety Domains at Different Target Bearing Angles**

| Bearing angle (degrees) | Distance (Elliptical domain), miles | Distance (Complex domain), miles |
|---|---|---|
| 0 | 3.20 | 3.00 |
| 30 | 3.17 | 3.18 |
| 60 | 2.29 | 2.45 |
| 90 | 0.80 | 1.00 |
| 120 | 2.29 | 2.28 |
| 150 | 3.17 | 3.08 |
| 180 | 3.20 | 3.00 |
| 210 | 3.17 | 3.18 |
| 240 | 2.29 | 2.45 |
| 270 | 0.80 | 1.00 |
| 300 | 2.29 | 2.28 |
| 330 | 3.17 | 3.08 |
| 360 | 3.20 | 3.00 |

The graph (Fig. 5.) constructed from these data clearly demonstrates that for both types of domains, the minimum safe approach distance has maximum values at bearings of 0°, 180°, and 360° (i.e., when the target is ahead or behind), and minimum values at bearings of 90° and 270° (target abeam). For the complex domain, the pattern of distance change is more smoothed, and the minimum values are slightly larger, reflecting the greater "width" of the safety zone in the transverse direction.

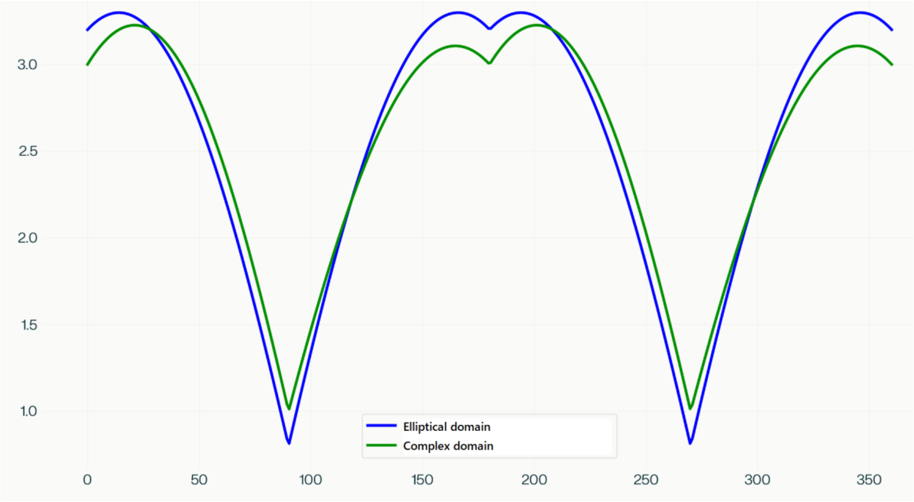

*Fig. 5*. **Dependence of Minimum Safe Approach Distance on Target Bearing Angle**

The graph shows two curves: blue - for the elliptical domain, green - for the complex domain. The horizontal axis represents the target bearing angle (from 0° to 360°), while the vertical axis shows the maximum allowable approach distance in nautical miles. Both curves have pronounced minimum values in the transverse sectors and maximum values in the bow and stern parts. This confirms the importance of taking into account the direction of approach when calculating the safe distance and selecting the optimal domain shape for different navigational situations.

Solving these tasks will contribute to improving the accuracy of assessing the level of danger during vessel encounters, which, in turn, will enable more effective selection of optimal evasive maneuvers. This approach will not only reduce the risk of emergency situations at sea but also significantly increase the overall level of maritime safety. Furthermore, the developed methods can be integrated into advanced intelligent collision avoidance systems, including autonomous vessel complexes capable of adapting to dynamic changes in the navigational environment and making decisions in real-time.

**Conclusions.** The analytical study of changes in the maximum permissible approach distance between vessels, taking into account the shape of the vessel domain, has resulted in new analytical dependencies that consider both the safe domain shape and the target aspect angle. The proposed expressions for calculating this distance for both elliptical and complex (combined) domains allow for flexible assessment of approach situations depending on the configuration of vessels' relative movement.

The research established that the nature of changes in the maximum permissible approach distance has a significant dependence on the approach direction, which is confirmed by graphical results of numerical modeling. The analysis showed that while elliptical and complex domains differ geometrically, the pattern of changes in the limiting approach distance remains similar in both cases. This indicates that in practical applications, both domain types can be used as tools for modeling safe space around a vessel, considering the chosen level of detail and available computational resources.

It was determined that using a complex-shaped domain allows for more accurate consideration of vessel maneuvering characteristics in various navigational situations. However, for practical implementation, it is advisable to choose a domain shape that provides an optimal balance between assessment accuracy and computational efficiency.

The obtained dependencies allow not only for quantitative evaluation of the permissible approach distance but also account for the influence of the approach aspect, which significantly affects decision-making by navigators or automated collision avoidance systems. The graphical presentation of results provided a visual demonstration of the distance dynamics depending on the angle between approaching vessels' courses, which can be used in the development of navigation system software.

These results can serve as a foundation for further improvement of collision avoidance algorithms and contribute to increasing the level of automation in navigational processes and overall maritime safety. The proposed approach allows for considering not only the absolute value of distance but also its dependence on the approach direction and safe domain geometry, which is crucial for modern intelligent navigation support systems, including those for autonomous vessels.

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## Адаптивне моделювання суднових доменів безпеки для запобігання зіткненням у морській навігації

*У цій статті представлено аналітичне дослідження змін критичної допустимої дистанції зближення між суднами, що наближаються, з урахуванням форми зони безпеки судна. Дослідження спрямоване на вирішення важливої проблеми забезпечення безпеки морської навігації шляхом розробки математичного підходу до точного моделювання зон суден за різних сценаріїв зближення. Запропоновано та виведено аналітичні вирази для розрахунку мінімально безпечних дистанцій як для еліптичних зон, так і для зон складної конфігурації, що дозволяє гнучко оцінювати ситуації зближення залежно від взаємного руху суден. Аналіз показує, що хоча еліптичні та складні за формою домени відрізняються геометрично, характер змін критичної дистанції зближення в обох випадках залишається подібним, що свідчить про можливість ефективного застосування будь-якої з моделей у практичних умовах залежно від необхідного рівня деталізації та доступних обчислювальних ресурсів. Графічне представлення результатів наочно ілюструє динаміку змін дистанції як функцію кута між курсами суден, що зближаються, що може бути використано при розробці програмного забезпечення для навігаційних систем. Отримані залежності дозволяють не лише кількісно оцінювати допустимі дистанції зближення, але й враховують вплив аспекту зближення, який суттєво впливає на процес прийняття рішень як судноводіями, так і автоматизованими системами уникнення зіткнень. Отримані результати створюють основу для подальшого вдосконалення алгоритмів уникнення зіткнень і сприяють підвищенню рівня автоматизації навігаційних процесів та загальної безпеки мореплавства, особливо в умовах інтенсивного трафіку або обмежених акваторій***.**

***Keywords:*** *домен безпеки, запобігання зіткненням суден, дистанція зближення, еліптичний домен, ухилення від зіткнень, ракурс зближення, управління рухом суден.*